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How can the science of complexity help young people realize their talents and choose their future?

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ABSTRACT

We live in a world of great discoveries in all sciences, but also serious threats to our very existence. This is, of course, not the first nor the last time we will be faced with such challenges, but every time the stakes are higher. Who can we turn to for help in such situations, if not the young people who now experience these threats and will have to struggle with them in the future? We all agree that education is our best hope. We also know that education is no longer an accumulation of knowledge, or an acquisition of aptitudes aiming to equip us with the means of securing employment. The explosive revolution in all realms of digital technology and access to information has changed everything. It is now, more than ever, imperative that the next generations develop a *horizontally broad education*, transgressing old boundaries, based on *interdisciplinarity* and a firm belief that, as Leonardo da Vinci noted, "*everything is connected to everything else*". In this paper, I will try to elucidate how the new science of complexity can help us achieve this goal, using several examples that can be used to infuse our youth with a true appreciation of all sciences and arts, excite their imagination and creativity, and enable them to face the difficulties that lie ahead. What, if not education, can accomplish that? When, if not now?

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Our motto in this paper is:
"Let's stop looking and start observing!"

Introduction

It is commonplace to argue that education plays the most important role in helping us improve ourselves, gain a clear understanding of the world around us, find our place in society and fully realize our potential to achieve personal fulfilment and satisfaction. What is not so "commonplace" is how to proceed in making sure these goals are achieved!

Let us start by agreeing, first, on what we mean by education: As is well-known, the ancient Greek word for this endeavor, "Παιδεία", comes from the word "παιδεύω", which is derived from "παις" (child). This refers to the act that Plato considered so essential for his ideal Politeia, as to propose that all children should be educated, independently of the influence of their parents' experiences and upbringing, to become effective citizens, able to undertake all responsibilities derived from fully participating in their civic duties.

Let us next state what I believe we all agree on: education is NOT the accumulation of information and the mindless storage of data, tucked away somewhere in our brain, so that it may "downloaded" upon demand. I am also confident we agree that Education must contain the elements of understanding the connections between different information items, no matter how disparate they may appear, so that we may perhaps be able to connect them in different ways and arrive at entirely different constructs from the ones we started with. Of all animals, this is an ability that only humans possess.

In so doing, we "educate" ourselves how to use the full realm

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of our mental and physical abilities to fully realize our potential and climb to peaks that we thought were previously unattainable. Thus, we will be able to understand what the young child, in Kazantzakis' book "Report to Greco", felt, when in response to his promise "I will reach as high as I can!" his grandfather replied, "No, I want you to reach where you cannot!"

So, we all know what education means. But do we practice it? This is where matters become difficult, as a major role is played by the education systems of our countries. In section "The Greek situation", I will speak about the situation in Greece, as it is my country, where I have taught at universities for more than 30 years, and, therefore, know best and care about deeply.

In section "The science of complexity", I will come to the central theme of this paper and outline how I believe the new science of complexity (Nicolis and Nicolis, 2007) can help us all, irrespective of our different education systems: i) rejuvenate our students' interest in all sciences, ii) make them appreciate the connections between science and the arts, and iii) induce them to think independently, ask questions and ultimately realize their own potential as future citizens in society.

In the following sections, I will discuss some of the wonderful new knowledge that complexity has offered us, its marvelous discoveries, and suggest innovative ways by which we can teach it to our youths. Thereafter, I will speak about connections of complexity and the arts, through the eyes of mathematics and physics and finally, the last section, I will offer some conclusions and suggest ways that I feel will be useful to educators, who would like to implement in their work ideas similar to my own.

The Greek situation

Let me start, for concreteness, by identifying different stages of our education system in Greece by 5 Cycles (Figure 1): Cycle 1 is Elementary Education, Cycle 2 is Gymnasium, Cycle 3 is Lyceum (or Lykeion), Cycle 4 is University and Cycle 5 is Graduate Studies. Now imagine these Cycles connected by "links", which pertain to how "effective" is the passage from one Cycle to the next. What does this mean? Clearly, I refer to how "smoothly" students pass from one Cycle to the next, having digested what they learned in the former and being able to easily follow the latter, without major "surprises".

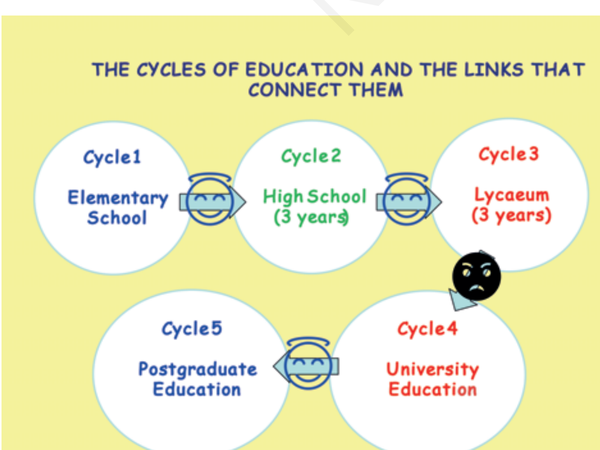

Figure 1. The 5 cycles of the Greek National Education system (author's design).

First the good news: In Greece, the link from Cycle 1 to 2 is well bonded. From Cycle 2 to 3, students "think" they understand matters, as there are no surprises, but still have only a general idea what they want to do in life and no clue as to what happens when they finish Cycle 3. They are generally quite influenced by what their parents and friends suggest, without seriously posing the questions: "What do I really want to do in my life?" "If science doesn't inspire me and have no obvious special talents, how can I best contribute to the betterment of myself and those around me?" "Should I become a very good technician, or could I perhaps work to expand my family's property, enterprise or traditions?"

I will now jump to the "link" between Cycles 4 and 5 and share with you the good news that matters are rather smooth here also, allowing for a "natural" transition. This is the "link" where I have devoted a great part of my life, with, allow me to say, reasonable success. But I prefer to further elaborate on this when I discuss my conclusions.

And thus, we arrive at the "weakest link", between Cycles 3 and 4, where the real problems in Greece arise. Here, there is no question of smoothness, but a case of disaster of great proportions, not only regarding the students' misorientation and their families' misfortunes, but also to the detriment of the wider Greek society.

To understand this modern Greek tragedy, one must take a close look at our National Education system, and how it was chiefly affected by a law passed soon after the elections of 1981, when Andreas Papandreou's PASOK party came to power. Although this law alleviated previous undemocratic practices of Professors identified with Chairs that did not allow the progress of younger faculty without the Professors' approval, it also contained some serious flaws, which soon became evident.

First of all, it kept all universities chained to a central National Educational System, whereby no university is allowed to form its own policies and determine the students it would accept in each of its departments. Everything was centrally organized, so that students have to pass National Examinations, stating their choices (mostly according to their parents' residence) so that admission grades became higher the more populated the city where each university is located.

But still, the worst was still to come. As the Entrance Exams became each year more elaborate, demanding students to answer more intricate and tedious questions, it became evident that what they learned in Cycle 3 was simply inadequate. Thus, a highly parasitic system of private institutions was formed, promising to teach the contestants exactly those intricate and tedious details needed to pass the Entrance Exams.

You can easily imagine what happened next: Lyceum education (itself strictly observing the Ministry's rules) became redundant, as it was less and less relevant to the students, in favor of the "recipes" they were memorizing at the private institutions. And so, all the beauty and appeal of Lyceum teaching subsided, compared with the utilitarian approach that what was important was what students "learned" privately, at the great expense of their parents, in whose eyes Lyceum education was also discredited.

"So what?" one might ask, as long as the students succeed in entering the Higher Educational and Technical Institutes closest to their choice and Exam grades. But, alas, this is not what happened. Forced, for several years, to recite what the private institutes "taught' them, the students forgot how to think independently, ask questions, and focus on what they enjoyed. Instead, they began to adopt a mechanistic way of parroting facts, which they immediately forgot when they entered Higher Education, so exhausted from this "training", that it often takes them 2-3 years to recover and start passing courses successfully!

And the "icing on the cake": In testimony to the harmful effects of the private institutes, recent data painfully show that the students' performance at the Entrance Exams has recently degraded so much that many University Departments have had to lower significantly the grades by which they will admit students, from say 60-70% to 40-50%, afraid to lose the student "clientele" assigned to them annually by the Ministry! Whatever you may wish to call this, it is certainly not education.

The science of complexity

As we all know, over the last 120 years, science has progressed with such great strides, as to have reached to date heights previously thought unattainable. From our understanding of the structure of elementary particles and the unification of quantum, electromagnetic, weak and strong interactions to the cure of life-threatening diseases and the complete deciphering of the human genome. We have also begun to understand how our brains function to the extent that we can now help invalids perform mechanical motions by electronically exciting the right area of their brain. We have also made progress in the direction of treating such dangerous ailments as epilepsy, schizophrenia, Alzheimer's and Parkinson's disease.

On the other hand, even though we have completely verified quantum physics and Einstein's theory of gravity revealing how our universe has developed since the Big Bang over the last 13.8 billion years, we still understand only about 5% of its constituents, as the remaining 95%, comprising what scientists call Dark Matter and Dark Energy, remain a mystery. Thus, as the great American scientist David Gross (Nobel Prize in Physics, 2004) recently said, "we may be proud of what we have discovered so far but let us not forget that our knowledge is finite, while what we don't know may indeed be infinite!"

So, the crucial question for Educators around the globe today is: how can we use all these remarkable discoveries to stimulate our students' interest, excite their imagination and infuse them with the desire to pursue themselves some of these incredibly attractive avenues of research or participate actively in their unnumerable applications?

This is where the science of complexity comes in (Nicolis and Nicolis, 2007): Complexity teaches us first of all, that living systems around us obey the very important rule "the whole is more than the sum of its parts", realizing group actions in ways that are impossible to explain by reducing our study to the analysis of their individual components. In other words, we may know everything about a single bird or fish, but this knowledge will never allow us to explain the incredible shapes the former make in the sky or understand the group changes of the latter in the sea (Figure 2).

You guessed it: if these strangely complex and beautiful group behaviours cannot be deciphered by knowing everything about their individual components, how can we hope to understand how our brains function, if we only understand the electromagnetic processes within each neuron and how they interact with each other? (Figure 3).

Remarkably, complexity science has an answer: instead of only focusing on individual entities, why don't we begin to study multi-node networks of them, endowed by arbitrary connectivity properties, and vary the "strengths" assigned to the links between each pair of nodes? Could we then hope that, by judiciously modifying these parameters, we may end up with a variety of complex networks that could exhibit a wide spectrum of unexpected group behaviors similar to what we observe in brain imaging?

You guessed it again: The answer is yes! Over the last 20 years, complexity has given rise to a new field called Network Science (Barrat and Vespignani, 2008, Latora *et al*., 2017). Its remarkable discoveries have revolutionized not only the way we study brain dynamics, identify diseases and model epidemic spreading (Loscalzo *et al*., 2017), but have also taught us how to comprehend social interactions (Vega-Redondo, 2013) and even optimize the performance of traffic networks (Chen *et al*., 2012).

Isn't this amazing? And isn't it about time we learned about it and began to teach it to our students? Wait a minute! You will

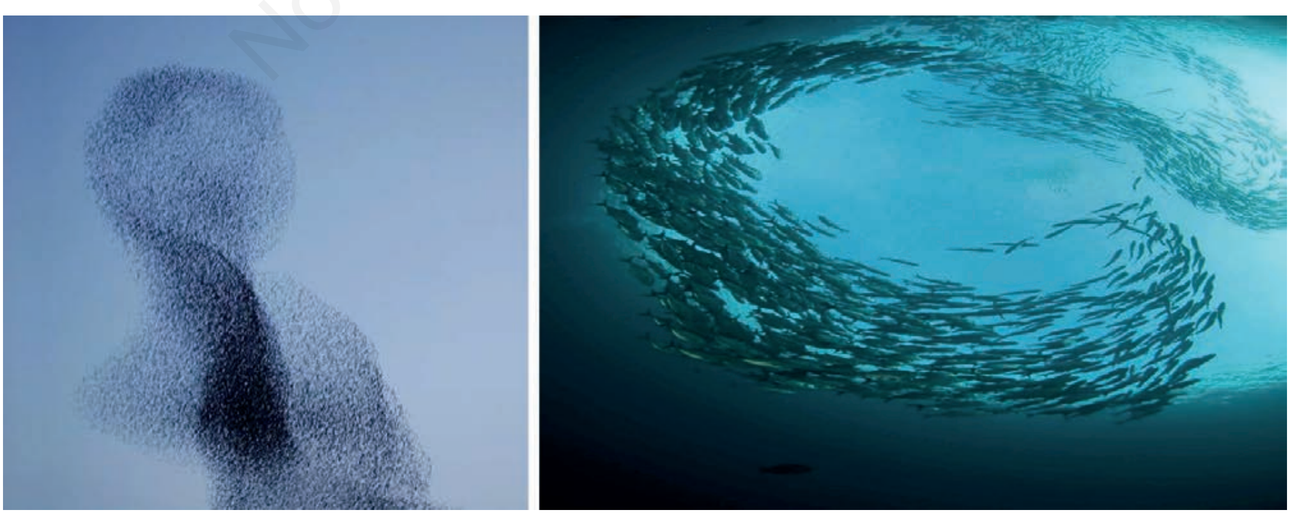

Figure 2. Bird formations (left) and fish "school" movements (right) cannot be explained by our biological and mechanical understanding of each bird or fish individually (photos publicly available).

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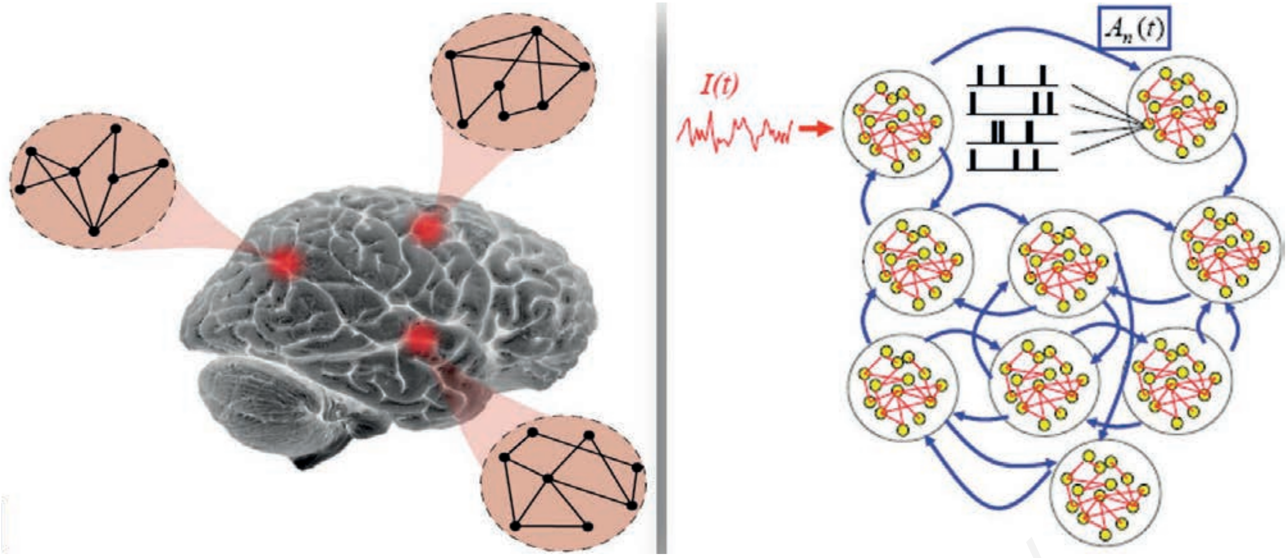

Figure 3. Considering the brain as a collection of "families' or subnetworks of neurons, we can model their activity on planar models consisting of such families of nodes, excited by outside oscillatory currents *I(t)* which can be analyzed into their Fourier components $A_n(t)$ (author's design).

protest. This material is too advanced and will require the knowledge of higher mathematics and computation, right? Wrong. Let me illustrate this with a simple example:

Let us take a large, say 100x100 square cells and mark each cell by anyone of 2 states: Red (R) or Black (B), (live or dead, whence the name Game of Life). Every cell interacts with its eight neighbors, which are the nearest cells horizontally, vertically, or diagonally to it. Now assign R and B randomly to all your cells and renew their colors successively by following a simple deterministic rule; at each step: i) any live cell with two or three live neighbors survives as R to the next generation (supported by its neighbours), ii) any live cell with fewer than two live neighbours dies and turns to B (say due to underpopulation), iii) any dead cell (B) with exactly three live neighbors becomes a live R cell (say due to reproduction), and iv) any live cell with more than three live neighbors dies, as if by overpopulation. This game was invented by the great mathematician John Conway in 1968!

Now continue to apply repeatedly the above deterministic rule to the new set of R and B cells and ask yourself what will happen in successive generations. Will the R cells finally dominate and "life" in this population will prevail, or will the B ones multiply to the extent that everyone "dies" in the end? Are there only fixed "states" as the game progresses, where the R and B cells oscillate in number? What do you think?

I will not tell you the answer, not because I don't want to spoil the story, but because… there is no single answer! The secret lies in your choices of the initial distribution of states! Depending on how you start, in general, you get a different answer! Now do you see what exciting developments we get when we mix a random process (initial choice of colors) with a purely deterministic one (evolution of the game)?

And thus, we come to what you have all heard as deterministic chaos or chaos theory, and of course the formation of fractal shapes also occurring through deterministic dynamical processes. Here allow me please to give only references to my books (since this paper is also addressed to many Greek educators), where all the theory is explained, as simply as possible, with many examples, since I have been teaching this subject since 1986 in Greek universities (Bountis, 1975, 1977, 2004; Bountis *et al*., 2017; Bountis and Mihailidis, 2019). In the bibliography of these references, you will find many of the well-known relevant books and articles that have appeared in the international literature.

So, let us now return to the main topic of this paper: what is complexity science? And, more importantly, what new concepts and principles does it offer, and how can we teach them to our high school students, to re-energize their interest in all sciences and arts, and help smoothen their passage through the "weak link" between Cycle 3 and Cycle 4 of their education?

How do we motivate students to learn about complexity science?

Let us begin with a simple-looking question: how do we explain the famous Zeno's paradox (of nearly 3000 years ago) that an arrow will never reach its target, since it will have to continuously halve its distance from the target? I suppose you know that the complete answer was actually given about 500 years ago when calculus and the theory of limits were formulated. But did you know that we can actually present this theory to our students using one or more …pizzas?

To clear the way, we first need to explain to the students that the there is a "confusion" in the paradox between the concepts of time and space (in fact, the theory of motion puzzled Ancient Greek philosophers for many centuries). The starting point here is that we should replace distances with time intervals. The paradox of "never reaching the target", after all, is related to a summation of time increments, which may be infinite in number but each of them is reduced by a scale r=½. Thus, what we really want to find is how to compute the sum:

$$\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)\sec = ? \tag{1}$$

assuming that the arrow travels first half the distance to the target in half a second (supposing its speed is equal to v=1 m/sec, but you can change v to any constant you wish). Thus, the next half-distance is travelled in 1/4 sec, the next one in 1/8 sec. *etc*., and the question is whether the sum in Eq. (1) is a finite or infinite number.

Let us forget about summing numbers and think about… eating half a pizza, then a quarter of the pizza, then an eighth of it, etc. Will this process of eating decreasing slices lead us to eat a 2nd pizza, then a 3rd, *etc*., all the way to infinitely many pizzas? Let's find out by a simple drawing in Figure 4.

Now I ask you: Is there any child above the age of 10 that cannot understand this? As I halve the slices continuously, add to the previous ones and keep doing this indefinitely, since the blank space (where the dots are) is always twice what I add, I will approach the top of the pizza in a continuous way and never surpass it! As the well – known Greek proverb says "ask a child or a madman to find out the truth!" (later in the Lyceum students learn that all this comes from a formula for geometric series but by that time it is too late).

Now that the children (and I hope you too) are getting interested, let us come to a more difficult question: suppose I come across a hungry child, whose appetite decreases more slowly, so that after half a pizza he (or she) eats 1/3, then 1/4, then 1/5 of pizza, *etc*. What will happen? Again, as any child (who knows about fractions) can verify 1/2+1/3+1/4 =26/24 and this means we need a piece 1/12 of a second pizza!

Let' keep adding: 1/12 (of the new pizza) + 1/5 +1/6 +… .+1/11 =1.02 approximately and we have finished the second pizza! Our friend continues to eat since, after all, the slices are getting smaller, and the bets begin! The shop owner promises a free 3rd pizza if we ever reach that point, whence, our young lad finds that this happens when he/she reaches the 1/29 slice. Now the stakes get higher, and a curious observer agrees to give our friend one euro for every new pizza he has to eat or get double his money back when we reach the last pizza. Do you think the child should accept the bet?

Well, our youngster is smart enough (like most children at that age) to use a calculator and try by a simple iteration algorithm to add up an increasing number of terms, with n=30, 50, 100, 1000…, in this so-called harmonic sum:

$$S(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\ldots\frac{1}{n}+\ldots \tag{2}$$

only to find out that S(n) continues to grow and exceeds any of the low integers of 4, 5,..,10,… or more pizzas! As we watch the numbers grow, we keep being fooled into thinking that S(n) tends to converge to some limit, only to find out every time that, after more iterations, whatever number we think of is eventually surpassed! But can we claim that we won the bet? Of course not! Our observing friend can always claim that had we summed more terms we could have reached a finite number.

I know of no better way to introduce a young student to the magic world of infinity, limits, convergent and divergent series, than the examples outlined above. In fact, even the concept of proof, that torments so many of our Cycle 4 students could well be taught here, in Cycle 3! There is a beautiful way to prove that the harmonic sum (2) goes to infinity and win the bet: It rests upon the well – known method of "reductio ad absurdum" that was already by the ancient Greeks and goes like this:

Let us *assume* that the sum (2) is finite. Dividing it by 2, therefore, yields:

$$\frac{S}{2}=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\ldots..+\frac{1}{2n}+\ldots. \tag{3}$$

Subtracting now (3) from (2) we clearly find for the other half the expression:

$$\frac{S}{2}=1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\ldots.. \tag{4}$$

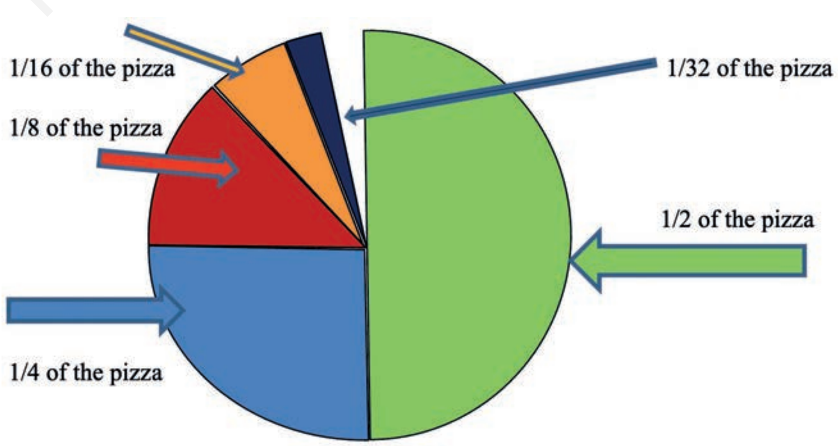

Figure 4. We can evaluate the infinite sum (Eq. 1) by eating pizza slices with ever decreasing size, which, as is obvious, never surpass a single pizza, but tend to eat all of the pizza in the end! (Author's design).

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It is also evident, however, comparing term by term the above two sums (3) and (4), that:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\ldots.. < 1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\ldots...$$

But that is impossible! How can one half be less than the other? We have reached an absurdity! The only possible explanation for this must be that our original belief that S is finite is false!

I am sure you can explain this argument to your students, and they will certainly understand it. Thus, you may end your class that day by saying: "Do you see now, boys and girls? Here is an example of how the human mind can discover what no calculator, however powerful, will ever be able to tell you!"

And a final bit of crucial information: Observe that the geometric sum (1) that gave us a finite answer involves terms which are powers of a scale r=1/2, while no such scale relation exists among the terms we added in the harmonic sum (2)!

The important lessons of self-similarity under scaling

I believe we can all agree that any new principle discovered is not important enough unless it can tell us something new about the world we live in. So, what can we say about the importance of this "scaling law" we encountered in the previous section? How can use the idea of scaling to understand something about nature?

To find out, we must first observe nature! Have you ever noticed something peculiar in the branching structure of a tree and the equivalent way the "capillary tubes" in every leaf divide themselves to provide water to the whole leaf (Figure 5)?

What is happening here? Observe first in the left picture that big branches "bifurcate" into two (or more) subbranches, which are significantly smaller in length (and width) from the "mother" branch. Now note a similar situation occurring as the leaf "tubes" "bifurcate" into thinner ones (right picture). The more observant among you will also notice that the distances at which the main leaf "tube" bifurcates in the second (smaller size) family are nearly equal, while the smaller distances, at which the bifurcations of the second ones also occur at similar nearly equal but smaller distances to give rise to the next "generation". Why do you think that is? Are there relative scales within the lengths and widths of these generations?

Let us try to answer the first question by drawing our own tree, taking a vertical length, calling it a, and repeatedly bifurcate from it two new branches at 45^o angles in an upward direction with lengths a/2, then a/4, *etc*., always at a scale r=1/2 smaller from one generation to the next (Figure 6, left panel). What do you observe? Even though we may repeat this process as many times as possible (practically infinitely many) the height of the tree will finally converge to a finite (relatively small) size! Could mother nature be employing such a scaling law for reasons of economy? You must admit it looks quite plausible.

Now let us turn to the leaf (Figure 5, right panel). Could the observed bifurcations between smaller and smaller "tubes", by some choice of scales, be what enables the nutrients to reach the furthest ends of the leaf, much like the arteries and veins in our bodies bifurcate to carry blood to the furthest extremities of our bodies?

Thus, according to our motto: "we have stopped looking and started observing!" And now nothing can stop us. Let us follow Barnsley's approach (Barnsley, 1993) and begin with any shape (square, circle, *etc*.), which will be taken thin and set up "upright". Next, we draw (Figure 6b) a second large copy of the initial shape above the thin one, tilted by a small angle to the right. Next, we draw a third smaller copy, tilted slightly to the left of the large one and a final thin parallelogram below all of them!

Now, let us iterate this process, using, over and over, the same simple geometric transformations that led us to the four initial copies of the original shape. The result is no less than a miracle! *No matter what the initial shape*, after many hundreds of iterations, we will always end up with the picture of the fern leaf

Figure 5. Note how smaller tree branches "bifurcate" from bigger ones (left) and how the "capillary tubes" divide as they carry nutrients to the ends of every leaf (right) (author's photos).

shown in Figure 6c! Isn't it amazing? What takes nature some months to create, we can reproduce it here in a few minutes of computation time!

This is a realistic application of the law of self-similarity under scaling, which complexity science teaches us. And now we can attempt to answer the question: Why does nature follow this law? Could it be perhaps that it would be extremely difficult to enclose so much information as pictured in Figure 6c in the small seed of a fern bush? Wouldn't it be a lot more economical to somehow "enclose" in the seed an "algorithm" of specific instructions how to create such a complex design?

Fascinating discoveries of complexity science

Thus, as we have finally begun to understand better the term "complexity", we may now become bolder and start looking for more important and useful applications of complexity science. Shall we begin by taking a closer view at how our heart functions?

At first sight, we notice that our heart behaves like a clock, which is quite similar to what the theory of dynamical systems calls a "limit cycle" (Bountis, 1997; Anastassiou and Bountis, 2019). As we know, this cardiac "clock" can increase its beating frequency (heart rate, HR) when we do strenuous physical exercise and reduce it when we are asleep. Under normal conditions, HR is quite steady at about 70-80 beats per min (Figure 7a).

Notice now that, besides the periodic polarization - depolarization "spikes" recorded by the EEG, there also appear aperiodic smaller "spikes" occurring chaotically at some of the electrodes. On the other hand, in the ECG of Figure 7), one notices more pronounced evidence of order and periodicity in the ECG signals. Unfortunately, in many cases, this type of "periodic" signals is associated with the case of ventricular tachycardia!

This demonstrates that nature prefers to endow the functioning of healthy organisms with a small degree of aperiodicity and chaos. Perhaps, in this way, a living organism becomes more

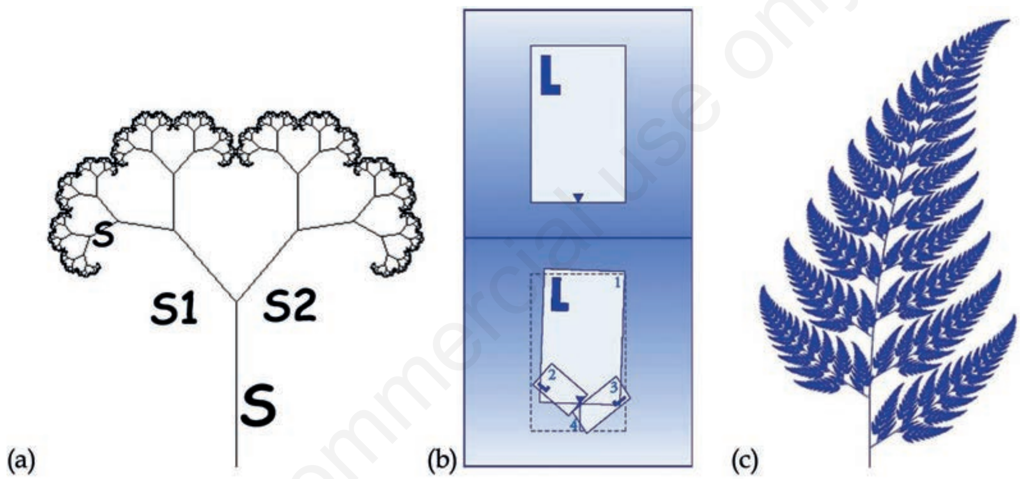

Figure 6. a) Construction of a mathematical tree by repeated decreasing of its "trunk" S by 1/2 and rotating by 45º to the right and 45º to the left (b) Employing repeated contracting transformations as those that relate the parallelogram marked with L at the top of (b) to the scaled copies 1, 2, 3 and 4 (a little stem below the others), we can imitate mother nature by creating a "realistic" fern in (c) (Bountis, 2004).

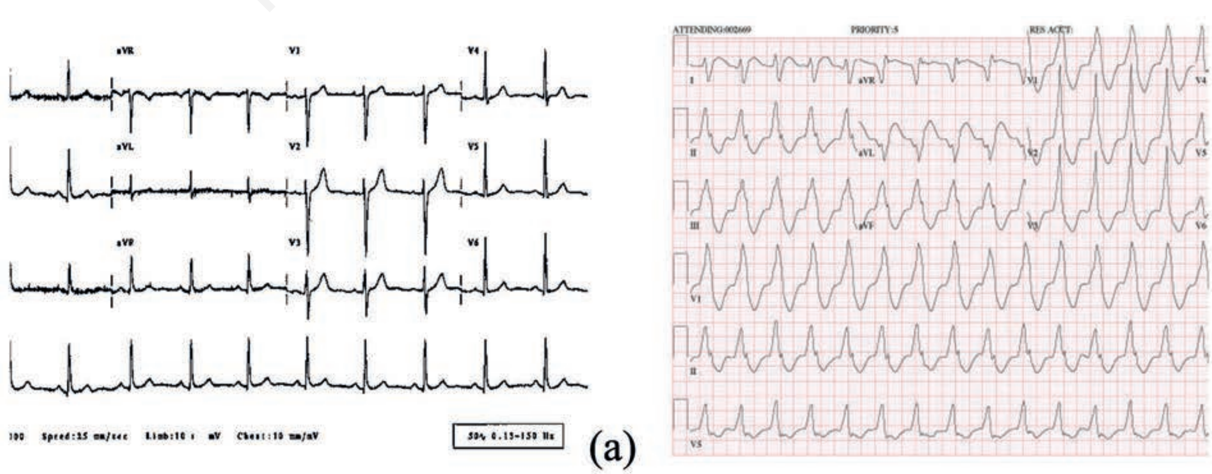

Figure 7. a) HR signal in the ECG of a healthy person under normal conditions. b) ECG of a patient suffering from ventricular tachycardia.

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"flexible" to "absorb" small "dosages" of irregular disturbances and thus adapt more easily to the surroundings without suffering undesirable effects.

This is also true of the brain! In fact, when studying the onset of frontal lobe epileptic crises, researchers have found that 10-15 min before the crisis occurs, the brain's chaotic behavior is significantly reduced (Iasemidis, 2003; Iasemidis *et al*., 2005), while immediately after the crisis, the electro-encephalo-graphic (EEG) signals show a remarkably periodic behavior (Figure 8). Thus, this discovery can be used, through a portable mechanism carried by the epileptic, to predict the onset of the next crisis, so that he/she can be warned and take the necessary precautions.

Now, we know that the brain works as a huge Neural Network (NN), consisting of many subnetworks, each responsible for a specific function (observation, motor, sound, language, *etc*.), which must "communicate" with each other to perform our daily tasks (Figure 3). Since neurons emit electromagnetic oscillatory signals, this communication takes place through the synchronization of these signals. This means that when there is synchrony the neurons of the communicating subnetworks oscillate synchronously, while when they don't, they exhibit asynchronous behavior (Haken, 2002; Salari and Maye, 2008).

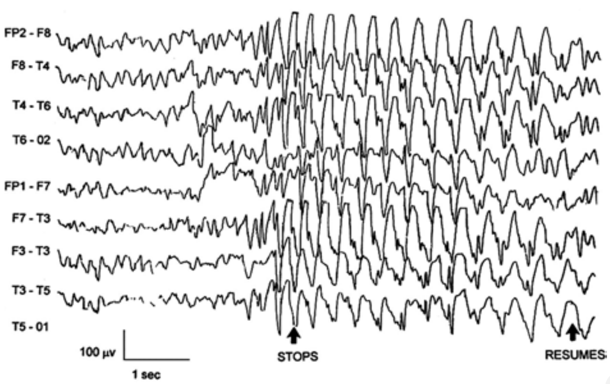

Figure 8. EEG of a patient suffering from frontal lobe epilepsy. The crisis starts at the point where regular function ceases ("STOPS") and normal function begins again at "RESUMES" (see Iasemidis, 2003; Iasemidis *et al.,* 2005).

So, let's take a look at Figure 8. Do you notice a weak type of synchronization in the left part of this EEG, where all the signals of the corresponding electrodes oscillate out of synchrony with a low level of "aperiodicity" or "chaos"? Good. Now observe what happens at the time where this activity ceases (designated by "STOPS"). This is where the epileptic crisis starts and lasts for several minutes until the previous state of weak synchronization and chaos is restored (designated by "RESUMES")! One cannot help but conclude, therefore, that synchronization is not always a healthy sign, while a healthy person's EEG must necessarily contain a low level of chaos.

Scientists were baffled for a long time about the fact that many mammals (including dolphins and seals) as well as many kinds of birds, often sleep with one eye open and one closed. This means, of course, that one hemisphere of their brain (corresponding to the open eye) is "synchronized", while the other one is "asynchronous". But how is that possible? Since the two hemispheres are connected through the neuron fibers of the corpus callosum?

As you know, scientists are never convinced they have understood something, unless they first build a mathematical model for it. So, a great wave of enthusiasm was generated when a paper appeared in the 2002 literature (Kuramoto and Battogtoch, 2002) establishing in a convincing way that such mathematical models of this behavior do exist. This was followed by a great number of papers, in which the name of "chimera state" was given for this a coexistence of synchronous and asynchronous oscillators (recall that *chimera* was a mythological beast made up by the combination of a lion, a goat and a snake; Figure 9).

To date great progress has been made in this direction, whereby hundreds of fascinating chimera states were discovered and verified in chemical and mechanical experiments. Of course, the most exciting prospect was to understand the brain, therefore, a great many discoveries followed studying more and more complex models of neuron networks demonstrating that such a

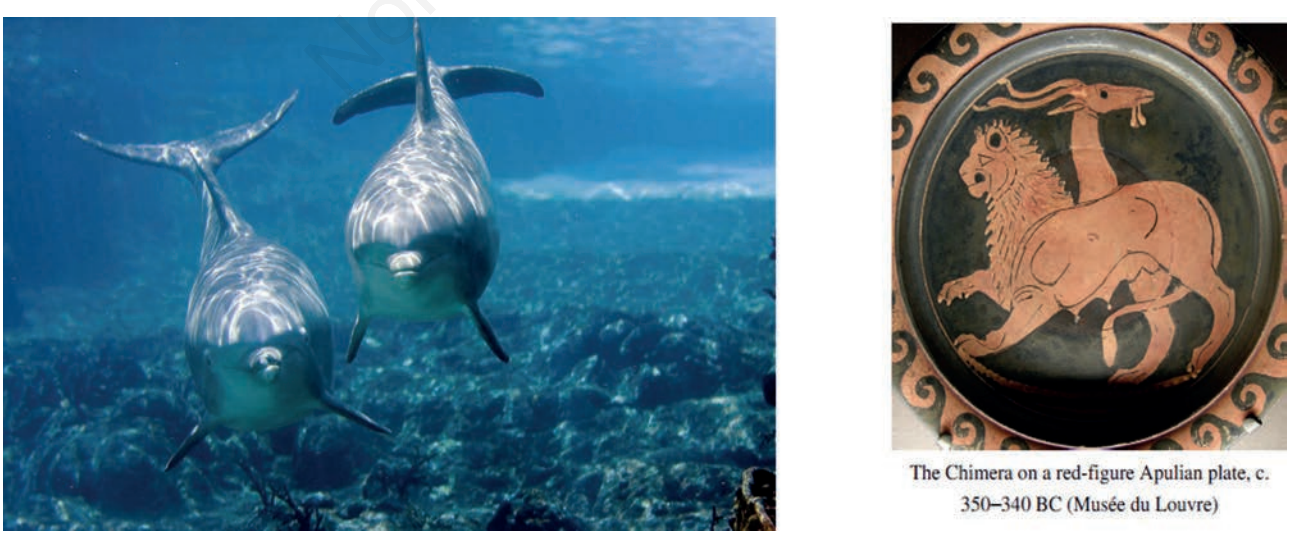

Figure 9. Left: some mammals and birds sleep with one eye closed and one open. Right: this combination of synchronous and asynchronous behaviour was termed "chimera state" after the famous mythological beast bearing that name (pictures available on the internet).

prospect is indeed realizable (see "3 Human Chimeras Already Exist", Scientific American, August 8, 2016, at: https://www.scientificamerican.com/article/3-human-chimeras-that-already-exist/). Here I will only mention a paper produced by our group at the University of Patras (Hizanidis *et al*., 2014), which has already received a significant number of citations in international literature.

From this paper, I reproduce here in Figure 10 one representative picture, showing on the left a column of snapshots of oscillatory states at different coupling parameters σ=0.005…, 1.0, where, at σ=0.005, 0.17 the asynchronous and synchronous neurons are homogeneously distributed, while at σ=0.02 and 1.0 all neurons are oscillating up and down in synchrony. A true chimera state is found at σ=0.47, where the different states are clearly separated.

Now let's become bolder and discuss something more advanced. In studying realistic mathematical models of neuron networks on the full set of 277 neurons of the simple *C. elegans* organism, we discovered that it was "naturally" divided into 6 distinct subnetwork communities (Antonopoulos *et al*., 2016), which often oscillated between synchronous and asynchronous behaviors.

To quantify the level of "cooperation" between these 6 neuronal communities, we employed (in Antonopoulos *et al*., 2016, see Figures 1 and 2 in that publication) a statistical quantity called Φ_{AR} (Massimini and Tononi, 2018), which measures the collective "response" between 6 the communities. If the value of Φ_{AR} is larger than the sum of the individual Φ_j of the j =1, 2,…,6 communities, we might conclude that the organism experiences a type of consciousness!

We found that this indeed happens when our parameters are chosen within the red and yellow regions, i.e., exactly at parameter values where the 6 communities are more strongly synchronized! Thus, we may conclude that synchronization is indeed an important phenomenon in these neuronal network models, and, moreover, it is precisely in synchronized domains of these models that we find evidence of "consciousness". What is needed presently is to perform actual experiments, for example through monkey brain studies, where these mathematical results are now beginning to be qualitatively verified (Hahn *et al*., 2021).

Complexity and the arts (is complexity beautiful?)

Who has not, at some time in one's life, looked up at a star-filled sky on a clear night and not marveled at the beautifully complex constellations, the distant stars of Sirius, Cassiopeia, Ursa Major and Minor or the planets of Mars and Venus and not thought of them as "beautiful"? And haven't we all heard from some visitors expressions of awe, while walking through the complicated arrangements of stalactites and stalagmites in a cave?

Even more so, when admiring the formations of birds in the sky flying over a dense forest landscape, or looking at the foliage of trees in a forest, why are so many of us fascinated by their "beauty"? I would understand finding all these sights "marvelously complex", but why "beautiful"? Does complexity have some aesthetic merits that we don't consciously realize?

To answer this question, it might be wise to look at the works of great artists, painters and sculptors for example, or lis-

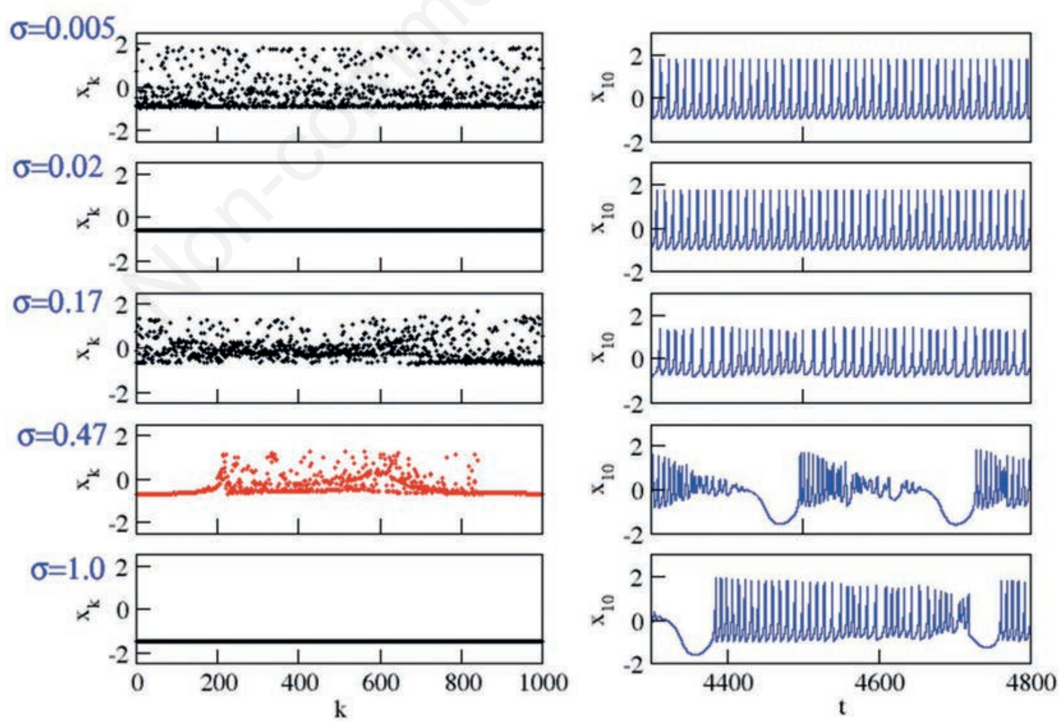

Figure 10. A picture from Hizanidis *et al*. (Int. J. Bifurc. Chaos 2014;24:1450030; with permission), showing different states on the left, only one of which shows a chimera state at σ=0.47 (see text for more details). Note that this state is produced by a specific pattern of high frequency "spikes" and low frequency "bursts."

ten to the music of famous composers. Aren't they supposed to reveal to us, common mortals, some of the inner beauty of nature by pointing out some of the "hidden secrets" of the world we live in?

I am sure you know of several such artists, only, up to now, you may not have fully appreciated the "complexity" of their creations. Perhaps you found them "interesting", "puzzling", or "peculiar", and went on to see other exhibits, or turned the dial to hear a different musical piece. However, I am sure you will agree with me that the more we know about a work of art, the more we understand it and even find inspiring.

Jackson Pollock (1912-1956) was an American artist, who painted large canvasses (like those shown in Figure 11), often stretched across the whole floor of his studio, using his famous "paint dripping" method (https://www.jackson-pollock.org/), and was hailed as one of the greatest painters of last century. In fact, he was the first great artist whose work was thoroughly analyzed from a complexity point of view by several scientists, who studied his paintings in terms of their self–similarity and found evidence of fractal properties, even measured their fractal dimension (Bountis *et al*., 2017 for a detailed discussion).

Following a similar approach, we analyzed (Bountis *et al*., 2017) some paintings of trees, by another famous artist of the 20th century, the Dutch Master Piet Mondrian (1872-1944). In particular, we chose the two paintings shown in Figure 12 and studied mathematically and computationally the fascinating fractal features of the trees appearing in them. More specifically, we focused on two parts of the trees' foliages and laid over them a frame of smaller and smaller "squares" of sides, say, L_1=0.75u, L_2=0.5u, L_3=0.25u, where u is a unit of length, e.g., 1 mm, depending on the size of the painting. (Figures 13 and 14).

For each of these situations, let us denote by N(Ln), n=1, 2, 3, the number of boxes that contain part of the tree inside them by first counting all the empty boxes and subtracting them from the total number of squares contained in the big rectangle. Next, we solve separately for each painting the following equations:

$$N_1(0.75u)^D=N_2(0.5u)^D, \qquad N_2(0.75u)^D=N_3(0.25u)^D$$

Figure 11. Two paintings, "Autumn Rhythm" (1950, left) and "Convergence" (1952, right) by the great American artist Jackson Pollock (1912-1956) (Pictures from the internet).

Figure 12. Two paintings, "The Red Tree" (1910, left) and "Farm Near Duivendrecht" (1912, right) by the Dutch artist Piet Mondrian (1872-1944), who lived earlier than Pollock (Pictures from the internet).

Eliminating from these equations the unit length u (which is irrelevant) and taking logarithms of both sides, we would expect that the values of D, D_1 and D_2 say, in the above equations would coincide, since they both lead to the same approximate measure of the tree M. This, of course, does not happen, since our first approximations are quite crude. It will be highly interesting, however, to ask if the following occurs:

Are the values of D_1 and D_2 close and lie between 1 and 2? As we move from the larger scales to the smaller ones, does the dimension converge to a single value D that would thus represent the fractal dimension of the tree in that region?

You guessed it: The answer to both questions is affirmative! To establish this convincingly, however, two scales are not enough. We need to use an iterative numerical algorithm to continue the above procedure to smaller and smaller scales that are not visible to the eye. When we did this (Bountis *et al*., 2017) we discovered that the values of D_n for n=3,4,… are indeed seen converge to the value D ≈ 1.75.

Next, we repeated this process for the painting "Farm Near Duivendrecht" (Figure 14). Applying the same procedure as above, we found that the fractal dimension of the part of the tree shown in Figure 14 is again D ≈ 1.75.

What does all this mean? Can we infer that there is fractal complexity in these paintings? Do our results mean that Mondrian was somehow attracted by these features and considered them important to apply to his paintings? Could we perhaps suggest that an analogous painting, whose creator is unknown, might have been painted by Mondrian, if its fractal dimension turns out to be close to D ≈ 1.75? And the ultimate question: is there some objective "beauty" in this kind of complexity, given that it did inspire such great artists like Mondrian and Pollock?

What do you think? I will not tell you the answers to these questions, as I am sure that, by now, you have the necessary knowledge to arrive at your own conclusions. I can only hope that what I have written here has provided you with enough means to form your own opinions. If you wish to discuss your views, even provide suggestions as to how they may be presented in the classroom, don't hesitate to reach me at tassosbountis@gmail.com.

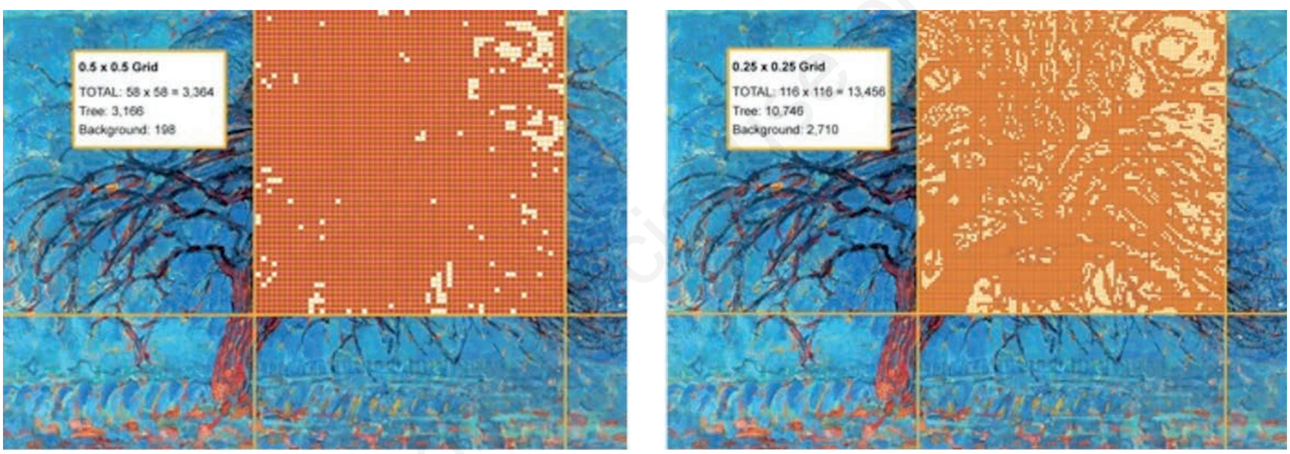

Figure 13. Two grid laying of part of the "The Red Tree". Left by squares of side L_1 and right by squares of side L_2 (see text). By dark red we have coloured in both pictures the squares that contain inside them any (small) parts of the painted tree and by light red squares those that don't. From Bountis *et al.,* Int. J. Arts Technol. 2017;10:27-42, with permission; Inderscience retains copyright of the article and figure.

Figure 14. Two grid laying of part of the "Farm at Duivendrecht". Left by squares of side L_1 and right by squares of side L_2 (see text). By dark and light red, we now color in both pictures the squares that contain inside them any (small) parts of the tree and by light blue squares those that don't. From Bountis *et al*., Int. J. Arts Technol. 2017;10:27-42, with permission; Inderscience retains copyright of the article and figure.

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Before proceeding to the next section and discuss the merits of applying concepts of Physics to study the history of art paintings, let me close this discussion by referring to some great composers of last century, who also produced "complex forms" of classical music. "Wait a minute!" you may protest. "What do you mean by 'classical' music?' Fair enough. I will give you a definition (which is not mine) but fully expresses my view: I shall call "classical" any work of Art, which, every time I run across it, I find something "new" in it, that I haven't experienced before. Isn't this, after all, what we have discovered to be true in all the complex forms of Nature we have encountered in this paper?

Let me finally point to a recent study, in which the technique of functional MRI brain imaging was used to show that, when mathematicians are exposed to musical or visual beauty there is activation in the same part of their brain (medial-orbito-frontal cortex), as when they are exposed to a "beautiful" mathematical expression, such as Pythagoras' theorem and Euler's equation exp(iπ)=-1 (Zeki *et al*., 2014).

The great painter Paul Cezanne (1839-1906) in his work *Girl at the Piano* (1850) alluded to Wagner's opera "Tannhauser", which was at that time the symbol of a new kind of art. Furthermore, the impact of music in Picasso's work was amply exemplified in a 2011 exhibition in the Museum of Modern Art, New York, entitled 'There is Music in Picasso and Picasso is in Music".

Finally, the widely acclaimed *atonality* in the music of the Austrian composer, music theorist, teacher, writer, and painter Arnold Schoenberg (1874-1951), has been related to works of his friend, the painter Wassily Kandinsky (1866-1944), hailed as one of the founders of abstract art (Dabrowski, 2003; Wright, 2007). Would you not agree that every time we look closely at the works of these masters, we discover something new? Isn't it about time we stopped looking and started observing?

Analyzing paintings through the centuries using concepts of physics

After discovering what mathematics can do to help us better understand the *complexity* of different paintings, it is interesting to ask whether other fundamental sciences, like physics, can be used to teach us something new, for example, about the way different *styles* of paintings have evolved *in time*, more specifically throughout the last ten centuries!

This formidable task was undertaken by the authors of a paper entitled "History of art paintings through the lens of entropy and complexity" (Sigaki *et al*., 2018). These scientists undertook the formidable task of studying paintings by 2000 artists, for 100 different styles, over 1000 years!

As a first step, each painting was converted into a matrix whose elements are the average values of the shades of red, green, and blue. Next, the authors used the probability distribution of 24 color patterns among the images to calculate two important physical measures: the normalized permutation entropy H (as defined in the paper by Bandt and Pompe, 2002), and the statistical complexity C=H•D (Hahn *et al*., 2021, for more details), where the entropy H quantifies disorder and D is a measure of "disequilibrium".

The results were remarkable and are shown in Figures 15 and

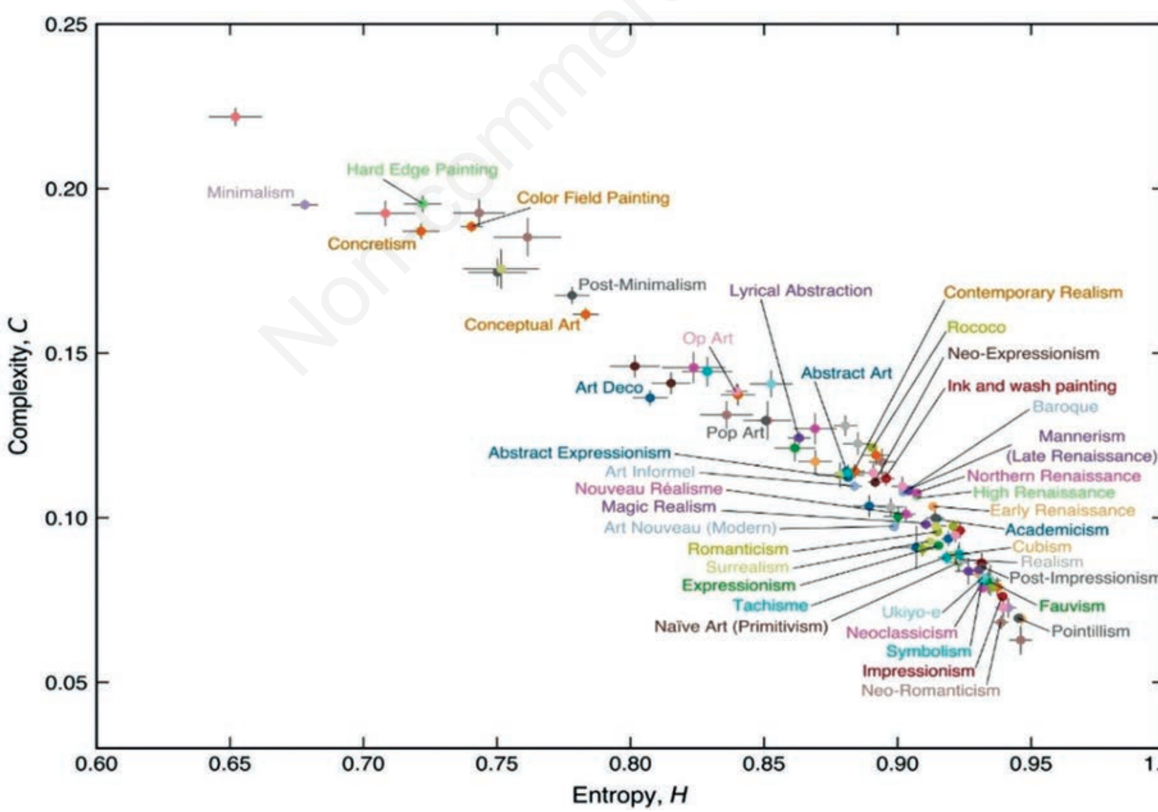

Figure 15. "Proximity" of different artistic styles in a complexity - entropy plane, of 41 artistic styles with nearly 500 paintings for each style. From Sigaki *et al*., P. Natl. Acad. Sci. USA 2018;115:E8585-94, with permission.

16. What do these figures tell us? Let's start with Figure 15, where we first observe that a great number of styles are grouped in the lower right part of the figure, characterized by low values of the complexity C and high values of the entropy H, while a group of much fewer styles appear in the upper left corner of high complexity and low entropy.

Many of the ones in the bottom half of Figure 15, such as Renaissance, Cubism, Expressionism, Romanticism, Fauvism, Pointillism, Art Nouveau, Classicism might be thought of as "related", but why is their complexity low and their entropy high? Could it be perhaps that their meaning is clear (low C), but their content is not very "ordered" (high H)?

And now look at the upper left part of the graph: I would understand that Minimalism, expressing simple geometric shapes, would be close to Concretism characterized by basic visual features, but why would their complexity be higher and their entropy lower than the other styles on the right part of the graph?

Finally, let us look at Figure 16: I find it interesting that Contemporary/Postmodern Art, on the upper left, and Modern Art, on the lower right, appear distinctly in the graph, in some agreement with the content of Figure 15. Interestingly, if we look closer at the box of the lower right, we find that it contains the dates 1939-1952 and 1902-1909, which are respectively close to the times when Jackson Pollock and Mondrian produced the "fractal" paintings we discussed in the previous section "Complexity and the arts".

The important question, of course, still looms, whether we are justified to use mathematical and/or physical quantities to study historically the evolution of the art of painting by analyzing the works of different artists, either individually or in groups. Based on what I described in the present section, I believe that such investigations are worth pursuing, as they shed new light on the different painting styles and their evolution in time and help us better understand the evolution of artists and their styles throughout the centuries.

What did we learn about education?

Thus, we arrived at the end of our journey. I hope you enjoyed it and feel you have learned some new fascinating knowledge. Now the time has come to ask ourselves, what does all this have to do with education, and how we may apply our new knowledge to improve current teaching strategies, as early as first year in High School.

I believe that all of you, as educators, have derived some ideas from this paper. Why not start, therefore, in your math classes, with the simple puzzles I developed in the section "How do we motivate students to learn about complexity science" about sums of decreasing numbers. Don't be afraid to teach the concepts of finiteness and infinity to your students. Beginning with the integers N={1, 2, 3, ….} explain to them the concept of countable infinities within the integers, and then turn to the fractional numbers p=n/m, where n, m are positive integers that have no common divisor and satisfy m>n.

Do your students realize that all these *infinitely fractions* are enclosed in the open interval between 0 and 1? Of course, they do. Can they count them? Easy: Number 1/2 by "1", {1/3, 2/3} as "2, 3", then {1/4, 3/4} as "4, 5", next {1/5, 2/5, 3/5, 4/5} as {6, 7, 8, 9} and so on. These numbers are called *rationals***.** Now pose to your students the question: can all numbers in (0, 1) be written in the form n/m? Let's find out: Note first that all these fractions, (or rationals) when written in decimal form are always composed of a finite sequence of digits, repeated *ad infinitum*.

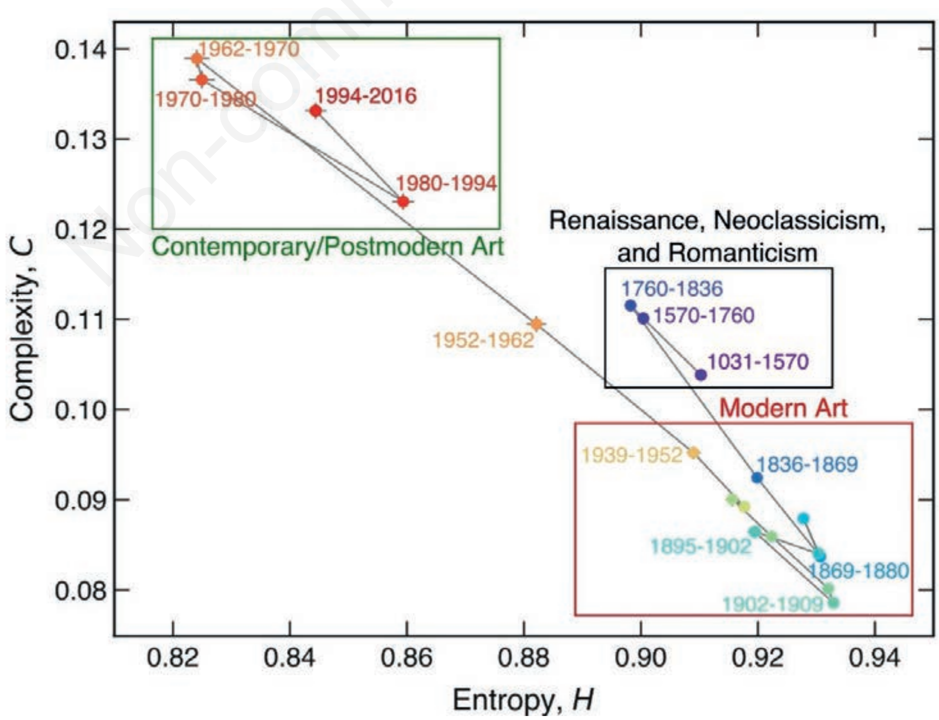

Figure 16. Grouping of different "art styles" through the history of painting, plotted on the H-C plane. Observe the difference in the years in each group! From Sigaki *et al*., P. Natl. Acad. Sci. USA 2018;115:E8585-94, with permission.

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Take, for example, the number 8/13= 0.**615384**6153846153846…. Clearly it is formed by the sequence 615384, which goes on forever, as the students can verify by applying the rules of simple division they learned in Elementary School. Now what about the number √2/2=0.70710678118654752440084436210485.... ? No matter how many digits a student computes on a calculator, he/she will never arrive at a finite sequence of integers that are repeated *ad infinitum*. These numbers are called *irrationals* and their properties cannot, of course, be taught at high school level. But, why not let the students experiment with them by themselves, using the internet to learn more about them? Don't you agree that this will allow them to generate their own questions and wonder about this puzzle on their own?

Now let's go to what we learned in the section "The important lessons of self-similarity under scaling" and focus on Figure 6. Refer the students to the literature, where all the steps for creating Barnsley's fern are stated explicitly. The students can then implement these steps on a simple computer program and construct their own realistic fern leaves! Not only that, but they can find in the literature other types of similar sets of simple transformations that will help students draw their own "Christmas trees", "ivy plants" (like those growing on walls), and many other fascinating designs, while being also able to create some new ones on their own!

This is what I mean by education. Don't give your students the full answer. Encourage them to think independently and search for the truth on their own, as far as they feel comfortable. This means you will have to do some extra work yourself, recall some of the topics you were taught at the University, but, believe me, it will be worth it! Similarly, if you are a physics teacher, don't hesitate to ask your students to find out more about galaxies, supernovas, red and white giants and "black holes". Tell them about the thousands of faraway exo-solar planetary systems discovered in the last 20 years, in which scientists are searching for "habitable zones" where Earth - like planets may exist that could support life similar to what we find on Earth!

If you teach Biology, what is wrong about encouraging your students to find out more about the human brain, its different functions, the imaging techniques developed to study it as a huge neuronal network, composed of many subnetworks that communicate with each other through synchronization? And believe me, if you start your students in that direction, there is no telling how far they may go!

Finally, talk to your students about the interrelationships between different sciences, by playing with them the Game of Life described in the section "The science of complexity". Ask them to explore the different patterns that arise using different initial conditions in the game. Starting from different arrangements of "living" (red) and "dead" (black) cells over a grid of NxN small squares (N= 10, 20, 50, *etc*.), encourage them to find cases where we don't end up with all cells of the same color. Can they "play God" and create communities of their own, which will sustain "life" forever? Will that "life" be "frozen" to a specific number of red and black cells, or will it be dynamically evolving "forever", with cells periodically changing color?

In this way, you will be able to motivate your students to appreciate the interdisciplinarity among all sciences: i) studying a mathematical model will make them think about its deterministic and statistical rules, ii) implementing it on a computer will help them understand how to write their own numerical program, while iii) interpreting the results will urge them to study more about biology and read how life was first generated and preserved on our planet. That's what I call education!

Now those of you who don't know me may very well ask: That's fine for you Dr. Bountis to give out all this "wise" advice, but what have you done to put into practice these educational ideas that you are preaching? Have you tried to explain advanced topics to young people to find out how effective your advice can be?

This is a very fair question that needs to be answered. First, I must admit that I haven't done anything significant to bridge the "link" between High School and University. I may have visited 10- 20 high schools in the Patras region, when invited by a teacher, or welcomed high school classes to the University of Patras to speak to them about chaos and fractals, but that was all.

Instead, I put all my efforts in bridging the gap between university education and graduate studies. I started, with many other colleagues, in 1987, a series of annual Summer Schools and International Conferences on Nonlinear Dynamics and Complexity that continues to this day. These 35 Summer Schools and 5 Conferences were attended by thousands of university Greek students, some hundreds of which went on to graduate studies in these fields and are today accomplished faculty members and researchers in Greek and international institutions.

If you wish to read more about these activities, go to http://cosa.inn.demokritos.gr/ and read chapters of my book on "The Meaning of Education: A Lifetime of Summer Schools".

However, I am still not satisfied, and now that I am a "retired" Professor, I intend to devote a lot of effort to help bridge this "weakest link" between High School and University in my country. If you wish to assist me in this effort, you know where to find me.

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