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Symmetry and Symmetry Breaking in Science and Arts*[1]

Klaus Mainzer

Carl Friedrich von Weizsäcker Zentrum, Universität Tübingen, Doblerstraße 33, 72074 Tübingen;
TUM Senior Excellence Faculty, Arcisstr. 21, 80333 München; European Academy of Sciences and Arts, St.Peter Bezirk 10, 5020 Salzburg

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mainzer@tum.de; klaus.mainzer@euro-acad.eu

ABSTRACT

In this review article, symmetry and symmetry breaking are considered as complementary principles in science and arts. It starts with symmetry and symmetry breaking in early world views of nature and art. Then, symmetries are definied as fundamental structures of mathematics. Mathematical models of symmetry and symmetry breaking are used to explain the emergence of space-time and matter in modern physics. Even molecular structures in chemistry are distinguished by mathematical symmetries. Their elegance and beauty seem to realize aesthetical categories in nature. In biological evolution, the question arises how symmetry breaking (e.g. molcular chirality) can be explained. In modern arts, symmetry and symmetry breaking are „hidden" structures which can be found in music, painting, and architecture. In a philosophical outlook, symmetry and symmetry breaking are highlighted as regulative guiding ideas of research.

KEYWORDS: *Platonic bodies, harmony, symmetry groups, automorphism, space-time symmetries, gauge symmetries, cosmological symmetry principle, global and local symmetries, spontaneous symmetry breaking, chirality*

[1] This article is an extended version and English translation of my article "Symmetrie und Symmetriebrechung. Von der Urmaterie zu Kunst und Leben," in: Schriftenreihe der Heisenberg-Gesellschaft: Quanten 2, 2014 (ed. Konrad Kleinknecht), S. Hirzel Verlag, 9-59. An abridged version was presented in the EASA Colloquium Science meets Art (https://www.youtube.com/watch?v=nIUVNEQ7AR8)

1. INTRODUCTION

Symmetries are used in the history of science and culture as fundamental models of order. This raises the question of whether they were merely invented by humans to order the diversity of phenomena, whether they even arise only from an aesthetic need, or whether they are basic structures of nature that exist independently of humans. In antiquity, at any rate, knowledge, art and nature were understood from a common symmetrical basic order. In modern times, this unity of natural and human sciences breaks down. In art, symmetries and symmetry calculations are related to subjective judgements of taste. In mathematics and the natural sciences, symmetries and symmetry breaking remain fundamental principles of describing nature, the application of which ranges from the formation of primordial matter to the evolution of life. In fact, current discoveries and laws in cosmology, physics, chemistry and biology are related to symmetry and symmetry breaking. It is symmetry breaking, according to the thesis in many of the author's books, that gives rise to diversity, complexity and new structures in nature - from physics and chemistry to biology and brain research. Without symmetry breaking, the world would remain invariant and unchanged. Mathematical structures make these interdisciplinary connections in nature and art transparent. (Johnson et al., 2022; Conway et al., 2008; Ball 2016). In this review although most analytical presentations have been summarized by eminent scholars from ancient Greece to the last decades, we present a summarized synthesis with due analytical reflections (Jonhson and Steinerberger 2019; Jantschi and Bolboac, 2020; Boi 2021; Tapp 2021).

2. SYMMETRY AND SYMMETRY BREAKING IN EARLY WORLD VIEWS OF NATURE AND ART

The search for patterns and regularities was and is vital for us humans in order to find our way in a jumble of impressions and signals from nature. Searching for patterns means reducing complexity. This is the origin of our search for laws with which we want to understand, explain and predict the processes in the world. Simple, regular and harmonious patterns have always been distinguished in order to reduce the complex and incomprehensible to them. To this day, therefore, symmetries exert a peculiar fascination on people of all cultures and religions. Whether it is the dome of the Hagia Sophia in Istanbul, the Taj Mahal in India or the rotunda of Aachen Cathedral - since time immemorial people seem to have wanted to represent the perfection of heaven with symmetries. In Judaism and Islam, where the divine may not be depicted as a person, particularly elaborate ornaments were developed. Occasionally, artists incorporated small deviations from symmetries into the ornaments, since perfect symmetry was reserved only for God and symmetry breaking defined the finite world.

Euclid's textbooks on geometry culminated in the proof (Mainzer 1980, p. 52) that there are exactly five regular solids in three-dimensional space, namely the cube of six equilateral squares, the tetrahedron of four regular triangles, the octahedron of eight regular triangles, the icosahedron of twenty regular triangles and the dodecahedron of twelve regular pentagons (Fig. 1). These mathematically fascinating solids made such a strong impression on Plato that he identified them with the then assumed elements of the universe: according to them, fire was made of tetrahedra, earth of cubes, air of octahedra and water of icosahedra. Later, the dodecahedron, made of pentagons, was added as the "quintessence" and building block of the celestial spheres. An ingenious idea was born: the universe, despite all its diversity, can be traced back to fundamental mathematical symmetries. This idea still dominates the mathematical description of nature today, for example in quantum and elementary particle physics. Today, simple geometric bodies are replaced by mathematical formulas on which the complexity of the world is to be traced back.

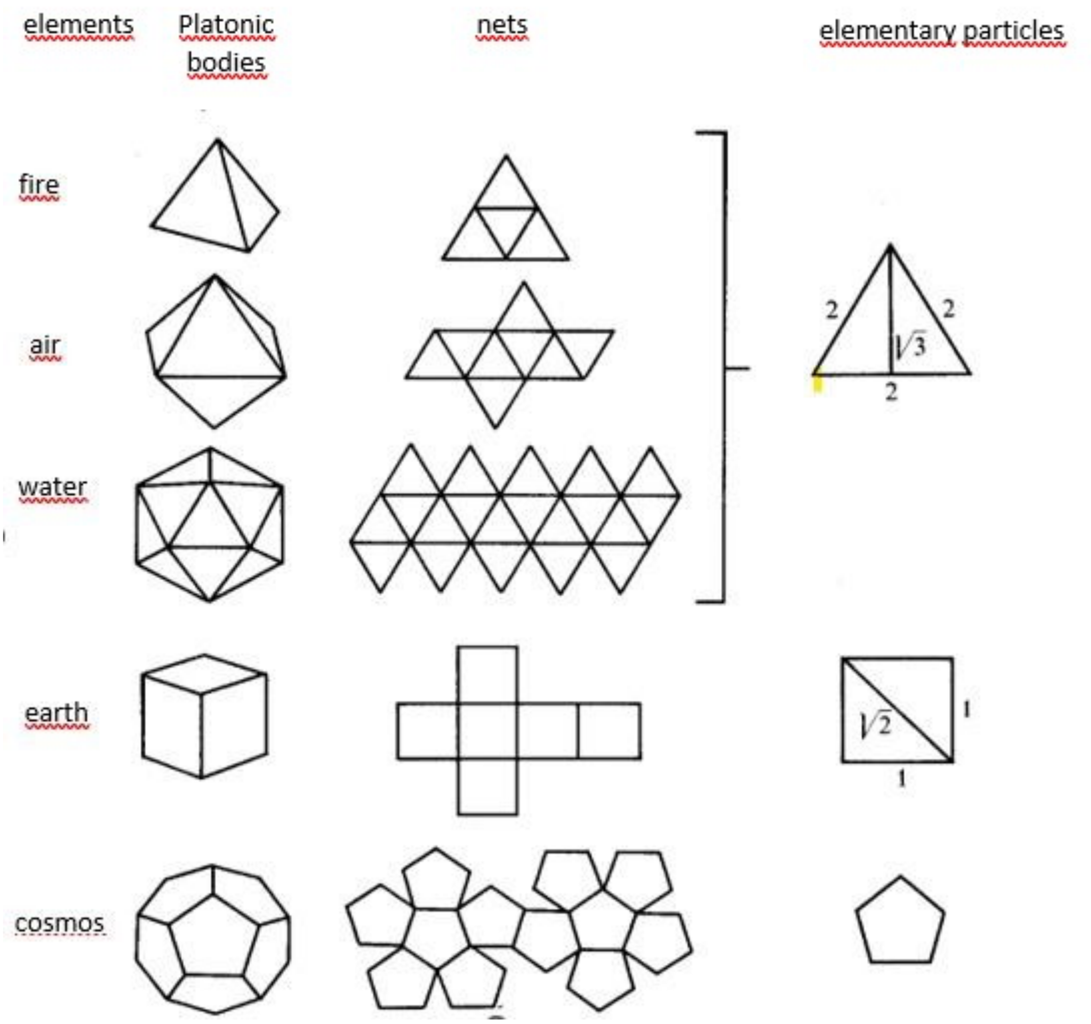


Figure 1. Platonic solids as a world formula

Symmetry existed for the Platonic world not only on a small scale, but also on a large scale: In the Platonic tradition, a centrally symmetrical planetary model is assumed - with the Earth in the centre, orbited by the then assumed convertible stars, to which the Moon and Sun were also counted. Here, the ancient connection between the natural order and art becomes clear. Apollo's lyre with its seven strings refers to the seven planets of Greek astronomy: the order corresponds to the sequence from the lowest to the highest note. The lyre conveys the music of the spheres of the planets and makes the cosmic harmony acoustically perceptible. Even Augustine (5th century AD) refers to music as the "sister of number" in the Platonic tradition. And then something monstrous happens for Plato: astronomers observe retrograde planetary orbits. This would be a symmetry breaking of the harmony of the spheres. For Plato, the observed retrograde planetary orbits (e.g. Mars along the ecliptic) could only mean appearances. Mathematicians had to think more deeply, look "behind" the external observations, so to speak, in order to "save the phenomena " (Mittelstrass 1970) and lead them back to the fundamental symmetry of the cosmos.

In fact, various geometrically exact procedures can be given to explain retrograde movements with geometrically uniform circular movements. One example is the exact construction of the ingenious mathematician and astronomer Eudoxos of Knidos, a contemporary of Plato. He assumes spheres centred in each other with differently inclined axes of rotation, so that the planet produces a retrograde motion loop on the equatorial circle of the outer sphere (Eudoxos 1966; Mainzer 1981). More flexible is the later epicycle and deferent technique dating back to Apollonius of Perga. According to this, all possible elliptical, regular, periodic and likewise non-periodic and asymmetrical curves can be traced back to uniform circular movements. The price here, however, is the abandonment of central symmetry: on a great circle (deferent) around the Earth, a uniformly moving point is assumed, which serves as the centre of a smaller circle (epicycle) on which the planet moves uniformly. In medieval astronomy, hierarchies of epicycles of epicycles "riding" on each other are assumed to produce all possible forms of motion.

At the beginning of the modern era, the belief in symmetry also inspired the great mathematician and astronomer Johannes Kepler. Thus he undertook systematic investigations of regular polygons and solids and was concerned with applications to crystals in nature. In his early work "Mysterium cosmographicum" from 1596, he even attempted to trace the distances in the planetary system back to the regular "Platonic solids". Here he already assumed a heliocentric world model in which the planets rotate on spheres around the centre of the sun. The planets Saturn, Jupiter, Mars, Earth, Venus and Mercury corresponded to six interlocking spheres, separated in this order by the cube, tetrahedron, dodecahedron, octahedron and icosahedron. Kepler's speculations could not be correct, because the discovery of further planets was reserved for later centuries. On the basis of more precise observations, Kepler finally abandoned his sphere model in favour of elliptical orbits.

3. SYMMETRIES IN MATHEMATICS

What is symmetry? In the early history of mathematics, symmetry (Greek *συμμετρία*) refers to the common measure or harmony of proportions of figures and bodies in art, architecture and the cosmos. Symmetry properties are, for example, reflection, rotation and periodicity. Mathematically, these illustrative examples are self-images (automorphisms) of figures and bodies in which their structure remains unchanged (invariant) (Weyl 2015; Mainzer 1988, chapt. 2). A geometric example of automorphisms are the similarity mappings, in which the shape of a figure remains unchanged (invariant) when it is enlarged or reduced. When automorphisms are linked (e.g. in a succession of periodic displacements or rotations), the structure is also preserved. Mathematically, all linkable automorphisms of figures and bodies are combined in a group.

The symmetry properties of a figure or a body are therefore uniquely determined by its automorphism group. Examples of discrete groups are the finite rotation groups of polygons that can transform a regular polygon into itself by finitely many rotations. If one takes into account not only the possible rotations of a regular polygon but also their possible reflections, the cyclic group is extended to the Dieder group, with which the symmetry properties of the figure are completely determined.

An example of a continuous group is formed by all continuous rotations of a circle by arbitrarily small angles, which always transform the circle back into itself. Continuous groups (named after the mathematician Sophus Lie) are of central importance for modern physics. Lie's idea of constructing homogeneous manifolds under the assumption of a continuous isometry group was generalised by Elie Cartan. Cartan understands by a "symmetric space" a Riemannian manifold in which the reflection at any point is an isometric transformation. For modern cosmology, "symmetric spaces" are of particular importance because they allow homogeneity and isotropy of the cosmos to be described on a large scale: In this case, no location (homogeneity) and no direction (isotropy) are distinguished, but always yield the same structure in reflections and displacements.

Further examples of discrete groups concern the symmetries of ornaments and crystals. The 1dimensional striped ornaments can be classified by 7 frieze groups, which are systematically generated by periodic displacements (translations) in one direction and reflections perpendicular to the longitudinal translation axis. If overlaps are also to be included in reliefs, both sides of a surface must be taken into account and the ornament list must be extended to 31 types. In 2 dimensions, 17 ornamental groups of the plane are obtained by translations in two directions, reflections, inversions and rotations. Historically, Kepler in his early work "Harmonice Mundi" (Book II) had already asked for the regular polygons that fill a plane. After partial classifications by C. Jordan (1869) and L. Sohnke (1874), the Russian crystallographer E.S. Fedorov provided evidence for the 17 ornaments in the plane. G. Pólya provided the group-theoretical classification in 1924. If the plane is additionally considered as a mirror, as in the case of the 1-dimensional ornaments, Pólya's classification can be extended to 80 symmetry groups.

Since the 19th century, symmetry properties have been used to classify various geometric theories. Following Felix Klein's "Erlanger Programme", one classified different geometries from the point of view of geometric invariants, which remain unchanged in metric, affine, projective, topological transformations. For example, the notion of a regular triangle is an invariant of Euclidean geometry, but not of projective geometry: it remains invariant to metric transformations, whereas a projective transformation changes the triangle sides.

The geometric transformation groups have considerable significance for the space-time concepts of modern physics.

4. SYMMETRY AND SYMMETRY BREAKING IN PHYSICS

Under the influence of mathematical analyses of geometric invariants, physical space-time concepts have been characterised by geometric symmetries since the 19th century. Thus, the Galilean-Newtonian space-time corresponds to an invariance of the classical equations of motion with respect to the group of Galilean transformations. What is meant by this is that, apart from Galilean invariance, equations of motion apply independently of uniformly moving reference systems (inertial systems) of an observer. So instead of the symmetry of figures and bodies, physics investigates the extent to which mathematical laws of nature are invariant to symmetry transformations.

For example, the laws of classical physics, such as Kepler's planetary laws, apply unchanged in all reference systems that move uniformly in relation to each other. They apply to Mars just as they do to Earth. If the coordinates in space and time are shifted according to the so-called Galileo transformations (Audretsch and Mainzer 1994, p. 38; Ehlers 1973), the mechanical laws remain the same. And because this symmetry applies everywhere, it is called a "global symmetry". In this case, the equations are insensitive to a uniform shift of all coordinates. Albert Einstein extended this symmetry consideration for his special theory of relativity by unifying the symmetries of classical mechanics with electrodynamics. With the new reference frame of the Lorentz transformations, global symmetry is also present here.

Analogously, the shape of a sphere is unchanging during a rotation if the coordinates of all points are changed by the same angle. Distortions or cracks occur on the surface if only "local" changes are made to the coordinates. However, the shape of the sphere is preserved if stretching and distorting forces are assumed, as with a rubber skin. With Plato, one could say that the symmetry of the sphere is "saved" by these forces because symmetry breaking by tearing the surface of the sphere is thus avoided.

In his general theory of relativity, Einstein worked for the first time with reference systems in which the global symmetry is broken. At some points in the space-time structure, local accelerations can suddenly occur. In order to maintain mathematical symmetry in the equations of general relativity, Einstein compensated for the local deviations by applying a force there: the gravitational force. With this force, Einstein's law of gravity remains invariant to space-time displacements despite the local symmetry breaks. This observation using gravitation as an example was later a useful heuristic for characterising quantum-physical interactions through "local symmetries".

In the standard model of relativistic cosmology, cosmic evolution is explained by the Cosmological Principle of H.P. Robertson and A.G. Walker. According to this principle, for an observer at any point in time, the spatial state of the universe is homogeneous and isotropic (given a suitable choice of scale). This symmetry principle can be determined mathematically by a continuous symmetry group in which the shape of physical quantities such as gravitational potentials and energy-momentum tensor remains invariant ("form-invariant "). The construction of homogeneous manifolds by isometry groups is a generalisation of the differential geometry of Riemann, Helmholtz and Lie, which was mathematically introduced in Cartan's theory of symmetric spaces.

In quantum physics, the electromagnetic interaction as well as the strong and weak interaction dominating between elementary particles can be determined by local symmetries of their laws: The interactions predicted by the theory do not change if one freely chooses certain quantities at a location ("locally"). This is reminiscent of the calibration of scales. This is why the mathematician Hermann Weyl spoke of gauge invariance or gauge symmetry when equations are invariant to arbitrary shifts of a quantity (Weyl 1918).

According to today's quantum-physical understanding of the fundamental forces in nature, there are mediator particles, bosons, for every force, which transmit the force and save the symmetry of the force equations. Thus, according to the understanding of quantum physics, the photon transmits the electromagnetic interaction. In 1954, the physicists Chen Ning Yang and Robert Mills developed a gauge theory that was to be used to describe the strong and weak interactions. It initially proved to be wrong because it assumed the corresponding mediator particles to be massless, as is the case for the massless photon. In fact, however, the bosons of the weak interaction discovered in 1984, for example, have a considerable mass. The range of the force they transmit is therefore finite.

Different mediator particles with different masses? This is again symmetry breaking and an obstacle if one wants to combine all the forces of nature in a Grand Unified Theory (GUT), i.e. in a set of formulas. However, the symmetry breaking of the particle masses can be corrected if one assumes that there is an additional mechanism that is invariant with regard to masses. This is the mechanism demonstrated by the Scottish physicist Peter Higgs in the 1960s (Higgs 1964, p. 132). It can explain why different gauge bosons have different masses. But Higgs theory itself requires a boson, the mediator particle for mass, so to speak. There is now much to suggest that the particle found by the European Nuclear Research Centre CERN is such a Higgs particle.

Cosmologists assume that all the fundamental forces observable today gradually separated from a uniform primordial force shortly after the Big Bang. There should therefore be an overarching formula composed of the splinters of the individual force formulas known today. In fact, at the beginning of the 1980s at the CERN research centre, it was experimentally possible to unite at least two of these individual forces: the weak and the electromagnetic interaction (Weinberg 1985). At very high energy, both interactions are no longer distinguishable. At low energy, however, this symmetry spontaneously breaks down. At even higher energy, it is physically explained how the strong interaction can also be united with the electromagnetic and weak interaction (Georgi 1980; Georgi and Glashow 1974).

Mathematical speculation, however, is still the primordial symmetry from which everything may once have emerged. Current approaches such as string theory do not yet offer any possibility of empirical verification. The confirmation of Higgs particles in 2012 already required a huge experimental machine like the particle accelerator in CERN. It is therefore to be expected that future progress will not only depend on theory, but also on even more precise measurement methods, increasing efforts in experimental technology combined with the growing computing capacity of supercomputers and the management of big data.

Theoretically, it would also be necessary to unify gravity with the three known quantum physical forces and their local gauge symmetries. Einstein's general theory of relativity would have to be merged with the quantum field theory of strong, weak and electromagnetic interactions. While Einstein's theory assumes arbitrarily small units in the sense of classical physics, the quantum world assumes a smallest size (Planck's quantum of action) on which the strong, weak and electromagnetic interactions depend. In descriptive terms, the quantum world is "granular", whereas the classical world is continuous. Classical physics is therefore conceived as an approximation to the quantum world: In the size dimensions of everyday life, one does not yet see the quanta of the microcosm and the world appears to be continuous.

Spontaneous symmetry breaking can also be found in everyday life. For example, an egg ideally has a completely symmetrical shape. Around the longitudinal axis, it looks the same on all sides. But if we place it with the tip on a smooth tabletop, it spontaneously falls to one side and thus breaks the rotational symmetry, although initially no direction was distinguished. Similarly, shortly after the Big Bang, the previously united forces could have spontaneously separated, and their exchange particles were each given different masses according to the Higgs mechanism, just as an egg tilts in a different direction each time it is tried several times. However, this was preceded by a symmetry described in grand unification theory (GUT) (Bernstein 1974).

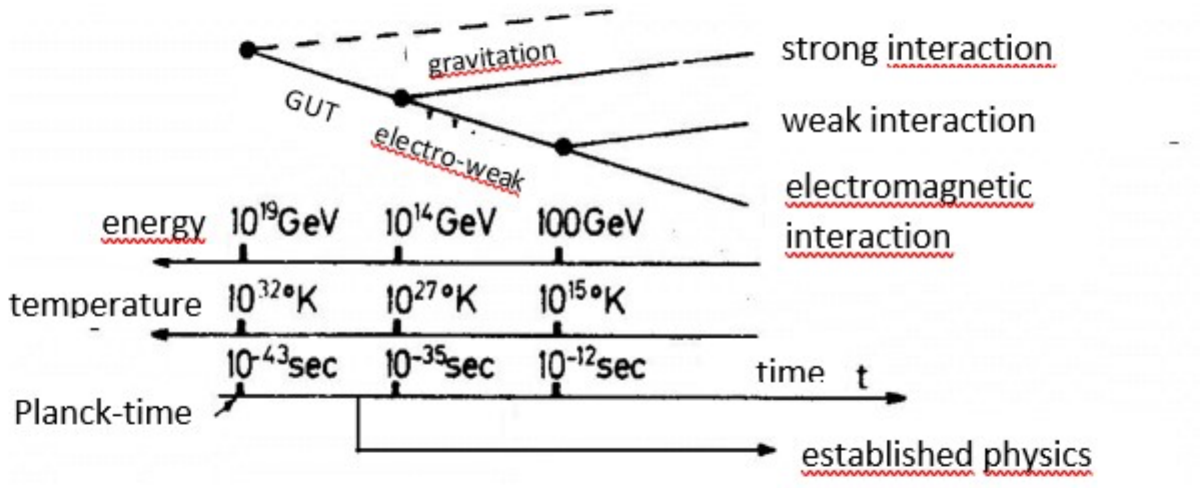


Figure 2. Spontaneous symmetry breaking and separation of the partial forces

In any case, the existence of today's universe can be explained by a series of symmetry breaking, in which the partial forces of the universe separated at critical states of energy and temperature during the expansion of the universe (Fig. 2) (Audretsch and Mainzer 1990, p. 98). This brings us back to the artists mentioned at the beginning, who incorporated symmetry breaking into their ornaments. This expresses a fascination for mathematical symmetries that is still shared today by researchers from different cultures. Whether the discovery at CERN is now a "God particle" may be doubted. In any case, it is another key to the high-energy laboratory of the universe in which we live.

Since Leibniz the conservation of energy in the universe was under discussion. In addition to mechanical energy, laws of conservation were proven for further quantities such as, e.g., angular momentum in mechanics. The question arose whether there is a relationship between laws of conservation and space-time symmetries. Under the influence of Klein's Erlanger Program, the mathematician Emmy Noether (Noether 1918) demonstrated how the 10 conservations quantities of mechanics follow from the invariance characteristics of the Lagrange function and Hamilton's action integral in relation to the transformations of the 10-parameter Galileo group. In general, Noether's famous theorem claims that, for a mechanical problem, there exist n conservation quantities if the equation of motion is invariant under an n-paramter continuous group of transformations of the (3+1)-dimensional space-time continuum.

On the other hand, we already know that the space-time of classical mechanics is determined by the Galilean principle of relativity, i.e. the equations of motion of a closed system of mass points are invariant under the transformations of the 10-parameter Galileo group. On the basis of Noether's theorem, therefore, the 10 conservation quantities of a closed mechanical system are completely defined. For the explicit calculation of the 10 mechanical conservation quantities of the Galileo group, the subgroup of space translations, rotations, time translations and translation velocities must be investigated.

From a historical point of view, Noether's results played a key role in the transfer of the mathematical concept of symmetry to physics. They were not only preceded in mathematics by Klein's „Erlanger Program" of geometric transformation groups, but also bei Sophus Lie's theory of continuous transformation groups. An additional important source was the algebraic theory of invariants which started in the mid-19th century with Arthur Cayley and James J. Sylvester, among others, who has been concerned with the calculation of „invariants" (discriminants) of polynomials.

It is remarkable that Noether did not only investigate the n conservation values which result from the invariance of Hamilton's principal function in relation to n-parametric Lie groups, but she also considered the invariance of Hamilton's principal function in relation to an „infinite-dimensional group" in Lie's sense with n given functions, which we now designate a gauge group. The mathematical context of Noether's theorem did not only make possible a complete determination of conservation laws of classical mechanics, but also of electrodynamics, relativistic gravitational theory, and, with appropriate modifications, applications in quantum mechanics and quantum cosmology (Schmutzer 1972).

5. SYMMETRY, ELEGANCE AND BEAUTY IN MATHEMATICS AND PHYSICS

The mathematical equations of quantum physics introduced in the 1920s were elegant and characterised by formal symmetries. However, these formulas are also abstract and can no longer be easily interpreted as intuitively as we are used to in classical physics. For example, the differential equations of quantum mechanics do not correspond to unambiguous motion curves of spheres as in classical mechanics. Rather, only probabilities for the future behaviour of elementary particles can be predicted. All possible trajectories and their respective probabilities must be taken into account in order to calculate the probability of a future event (Audretsch and Mainzer 1996). A brilliant mathematical physicist like Paul Dirac therefore believed that exact formulae without intuitive interpretation were sufficient to calculate the quantum world.

Richard P. Feynman (1918-1988) belonged to a younger generation of physicists and was not satisfied with abstract formulae. Unlike conventional computers, humans work with visualisation and images which are the origin of human aesthetic sensitivity. This has to do with our visual abilities, which have been highly developed in evolution. With a great sense of didactics, Feynman proposed intuitive diagrams for quantum electrodynamics that illustrate collisions of particles. Such a diagram shows the initial state of the incoming particles and the final state (the reaction products) as well as all collisions that still take place in between but are not directly observable. Fig. 3 shows only one of many possible detailed diagrams. In any case, each of these illustrative pictures stands for a mathematical term. By multiplying these terms, one obtains the probability with which the interaction takes place.

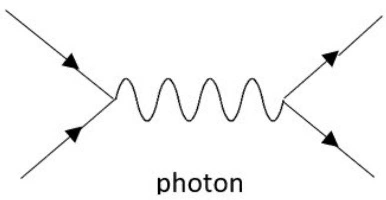


Figure 3. Illustration for formulas by Feynman diagrams

Whole generations of physicists have followed the suggestion of these Feynman diagrams to this day. They do not actually explain anything, but help us humans to find our way through a highly abstract mathematical apparatus and complicated calculations. Feynman's generation of physicists was inundated for the first time by a flood of data from high-energy physics. As a young physicist, Feynman had already participated in the construction of the atomic bomb in the Manhattan Project. The importance of data and the crucial role of engineers will have been impressed upon him. Indeed, his diagrams are reminiscent of the circuit diagrams of electrical engineering. They, too, can be directly translated into differential equations that can be used to calculate current data. Later, research reactors and particle accelerators were added in the USA, with which new elementary particles were discovered all the time.

In the 1950s and 1960s, a wealth of new particles were discovered that interact with the strong force and are therefore called hadrons (Greek for "strong"). With more powerful particle accelerators and energies, more and more hadrons could be created. The discovery and study of these particles became dependent on the state of development of high-energy technology. However, a fundamental theory of the strong force was far away at that time. In the history of science, the physics of strong forces showed a remarkable development pattern. In the beginning, there was "big data" with an immense variety of disordered measurement data and particle discoveries, which virtually led to a "zoo of hadrons". In a second phase, commonalities, analogies and correlations were noticed, which led to ordering schemes. But they were largely only approximations and not exact physical explanations. This refers to the so-called charge multiplets (Itzyson and Zuber 1980, p. 513), with which particles were arranged according to their charges in geometrically illustrative ornaments (Fig. 4).

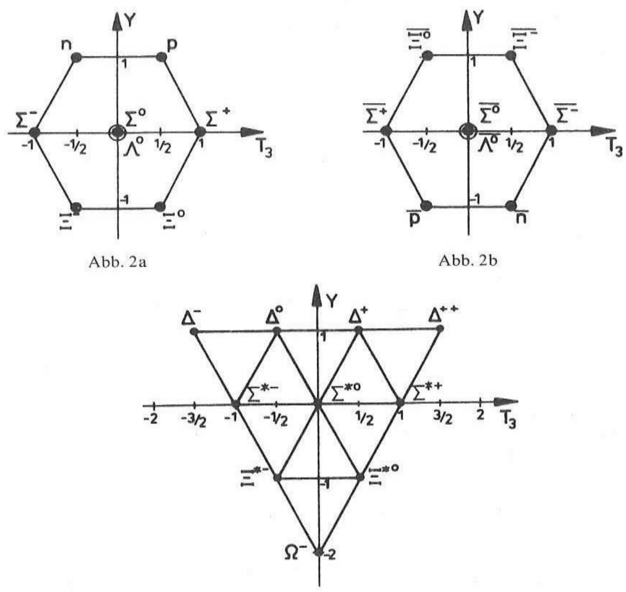


Figure 4. Symmetries of particle multiplets

Occasionally, these images led to the discovery of new particles. Despite these heuristic successes, the particle multiplets seemed to many physicists like the mystical symmetries of Kabbalistics, whose simple rules of combination could be learned but whose rationale remained obscure. Murray Gell-Mann and Yuval Ne'emann took a mathematically important step in 1962 when they were able to describe mathematical symmetries in these ornaments using mathematical group theory. Alluding to Buddhist wisdom teachings, they also spoke of the "eightfold path" that is necessary to recognise the underlying symmetry behind the multiplicity of particles (Gell-Man and Ne'eman 1964). Cultural traditions and aesthetic inclinations may not play a role methodologically for the development of theory. In the psychological background of the research process, however, they often operate in an unconscious way and must therefore not be ignored when considering the entire research process as a cultural achievement of human beings (Doncel et al. 1987).

Nevertheless, some properties remained unexplained and the multiplets at best think-economic, aesthetic and approximate descriptions in the Big Data of the Hadron Zoo. The decisive explanation was given by GellMann and George Zweig in 1963 with the proposal to trace all hadrons back to a few elementary building blocks. For these particles, known as "quarks", it is indeed possible to specify exact fundamental symmetry groups with which the Big Data of hadrons can be precisely calculated and predicted.

Quarks and leptons are fundamental building blocks of matter (Commins and Bucksbaum 1983). Both are fermions, because they have half-integer spin and follow the Fermi-Dirac statistics. The standard model of elementary particle physics distinguishes six quarks such as up (u), down (d), charm (c), strange (s), top (t), and bottom (b) and six leptons such as electron (e), muon (μ), tau (τ), and their corresponding neutrinos ν_e, ν_μ, ν_τ. Each fermion has an antiparticle with the same mass and lifetime but opposite charges (e.g., electromagnetic charge, weak charge) of the particle. From a symmetric point of view, antiparticles are mirrored by a transformation that changes all the charges. The standard model assumes that all particles interacting by the strong interaction consist of the six quarks such as, e.g., a proton p made of two u quarks and one d quark with $p = uud$ and a neutron n made of one u quark and two d quarks with $n = udd$ (Mainzer 1988, chapter 4.33).

The six types u, d, c, s, t, b of quarks are called „flavor" of the quarks. The term „flavor" does not relate to the taste of human senses. Flavor symmetry concerns relations between hadrons that consists of flavor quarks (Perkins 2000, Groom 2000). These relations exist, because the strong force which binds quarks into hadrons acts with the same strength on all quarks independent of their flavor. But these relationships are only approximate. The reason is that quarks must be distinguished as light and heavy with respect to their masses compared to the mass of a proton. Flavor symmetry is appropriate for hadrons which consist of light quarks such as up, down, and strange quarks. In this case, the masses of light quarks differ less from the mass of the proton. For all heavy quarks such as charm, bottom, and top quarks, flavor symmetry is no longer a good approximation. The flavor symmetry of up and down quarks is an example of good approximation which is known as isospin of SU(2) symmetry. The flavor symmetry of all three light quarks is a less good approximation. At first, the SU(3) symmetry of hadrons was introduced by Murray Gell-Mann and Yuval Ne'eman to classify properties of a variety of observed particles. Later on, the quark hypothesis of Gell-Mann and Zweig assumed quarks which should explain the observed SU(3)-symmetry.

The strong interaction between quarks is mediated by so-called gluons. They are the carriers of the strong interaction, such as photons are the carriers of electromagnetic interaction in quantum electrodynamics. The analog to electric charge is called „color" of the quarks. Therefore, the quantum field theory of strong interaction is called quantum chromodynamics. The three kinds („colors") of charge in quantum chromodynamics should only roughly remind of the three kinds of colors „red", „green", and „blue" in human perception. Mathematically, quantum chromodynamics is a non-abelian gauge theory (or Yang-Mills theory) with symmetry group SU(3).

The processes between quarks and leptons are described by the three symmetry transformations charge conjugation *C*, parity transformation *P*, and time reversal *T*. In a charge conjugation, all particles of a system are replaced by their antiparticles with all charges changing sign. This symmetry is only true for neutral particles, which do not carry any charge. The parity transformation corresponds to a space inversion relative to a point. Time reversal transformation means inversion of the time variable. The quantum theory of fields which describes quantum processes compatible with special relativity satisfies the invariance of the fields and interactions under the combined transformation *CPT* of the three transformations. The *CPT*-theorem (Lüders 1954, Pauli 1955) has been confirmed in experimental tests (Greenberg 2002).

In 1956, Lee and Young examined the question whether processes driven by the weak interaction would distinguish left from right (Lee and Young 1956). The famous experiments in beta decay of ^{60}Co and in weak decays of pions and muons showed that parity violation is a universal property of the weak interaction. As consequence of the *CPT*-theorem, the violation of *P* requires the violation of one of the other two symmetries. The charge conjugation transformation was also violated in these measured decays. Therefore, the parity violation of weak interaction is one of the fundamental properties of unified theory in the standard model.

6. MOLECULAR SYMMETRY AND AESTHETICS IN STEREOCHEMISTRY

In natural science, chemistry is understood as a bridge between the micro- and macroworlds (Mainzer 1988, chapt. 4.41). It deals not only with electrons, atoms and molecules, but also with macroscopic substances and objects. While historically people were already familiar with the spatial shape of macroscopic phenomena from everyday life, the idea of a spatial structure of atoms and molecules in chemistry seemed by no means self-evident. For atomists in the wake of Democritus, atoms were small hard objects that combine in empty space. Even John Dalton assumed atomic spherical shapes as the basic building blocks of matter. After chemists had learned to distinguish between molecules and atoms, the question arose as to how the structure of molecules from atoms could be imagined spatially. Crystallographers such as Bravais initially assumed that crystals were composed of small regular building blocks, without thinking of atoms. A hint of a possible 3dimensional structure of molecules from atoms can be found in Leopold Gmelin's "Handbuch der theoretischen Chemie" [Handbook of Theoretical Chemistry] as early as 1847. The decisive impetus, however, came from Louis Pasteur's experimental investigations into the optical activity of tartaric acid.

Linear polarisation was known in geometrical optics. With a Nicol prism, the oscillation plane of an electromagnetic field could be fixed. It was shown that certain substances, which were called optically active, turn the plane of oscillation of polarised light to the left or right. In 1822, John Herschel investigated mirrorimage quartz pairs, one of which rotates the plane of oscillation to the left, the other to the right. This proved a relationship between the mirror symmetry of the crystal structure and optical activity.

Pasteur recognised that the connection between reflection symmetry and optical activity does not depend on the crystal structure of a substance. In certain water-soluble crystals, the mirror symmetry can be demonstrated in both the solid and liquid state. Pasteur investigated tartaric acid and found a levorotatory and a dextrorotatory form called L-tartaric acid and D-tartaric acid (D=dextro=right) respectively. In addition, he isolated a third tartaric acid (meso-tartaric acid) that could not be split into one of the two levorotatory and dextrorotatory copies. To explain the optical activity, it was therefore necessary to resort to structures deeper than the crystals, namely the molecules and storage of the atoms. Another important step was August Kekulé's investigations on 4-valent carbons, for whose multiple bonds he introduced a notation of the structural formulae that is still common in organic chemistry today.

Decisive for the assumption of a 3-dimensional molecular structure, however, was the work of Jacobus Henricus van't Hoff and Joseph Le Bel, who in 1874 independently of each other established a relationship between the optical rotational capacity and the position of the atoms in space. The initial example was the carbon atom, whose four valences were arranged in the form of a tetrahedron. A tetrahedral arrangement with the carbon atom in the middle enables the existence of optical mirror images. In Fig. 5, the left tetrahedron arrangement cannot be made to coincide with the right one. Below is the 2-dimensional Fischer projection of the tetrahedra. In the case of tartaric acid, there are two identical carbon atoms, each of which is connected to the atoms or atom groups H, OH and COOH. For this composition, there are two layers (L- and D-tartaric acid), which are mirror images, and one layer (meso-tartaric acid), which is symmetrical within itself (Fig. 5).

Van't Hoff's stereochemistry about the spatial construction of atoms had to appear at first as a highly speculative idea, betraying a certain proximity to Platonic ideas. Kekulé was possibly particularly trained in spatial perception through previous architectural studies. In terms of the history of science, it is noteworthy that in the second half of the 19th century, parallel to stereochemistry in mathematics, a fruitful epoch of geometry and algebra took place. Van't Hoff's successes in experimental explanation and prediction soon made his geometry and algebra of molecules the accepted method of the chemist. But it lacked a definitive physical justification. At this stage of development, stereochemistry remained a successful research method that met the chemist's need for clarity in his structural analysis.

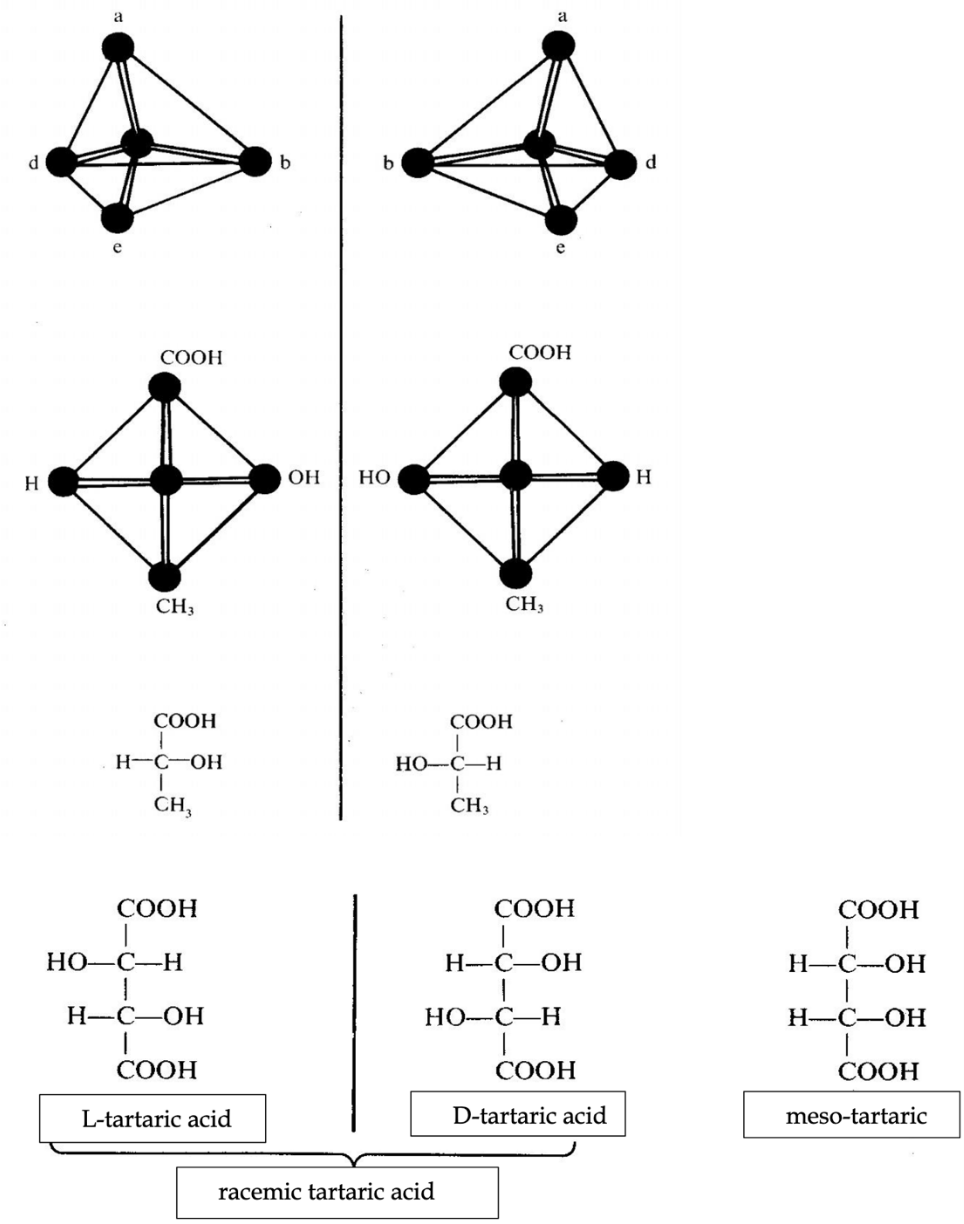


Figure 5. Mirror symmetry and molecular structure

A physical justification of stereochemistry was first provided by modern quantum chemistry. The historical beginning of quantum chemistry falls in 1927, when shortly after the publication of Erwin Schrödinger's wave equation, Walter Heitler and Fritz London as well as Max Born and Julius Robert Oppenheimer published two fundamental papers on the chemical theory of molecules. Methodologically, the question is whether and how molecular stereochemistry with its classical models of spheres and rods (Fig. 5) can be obtained from the principles of quantum mechanics. This gives the symmetry properties of molecular structures a physical justification and they are no longer just properties of merely expedient models. In fact, molecules, electrons and positrons are quantum systems that are entangled with each other and cannot be isolated locally. In principle, a full quantum mechanical description of molecules is conceivable and physically correct. However, this does not determine the chemically relevant properties. Familiar chemical terms such as molecular structure, nuclear framework, oscillation and rotation are quasi-classical and are only introduced by the abstraction of the chemist according to the Born-Oppenheimer method. Here, the nucleus and electrons of a molecule are determined as separate objects of a molecule. In the asymptotic limit case, a rigid nuclear framework of the molecule is derived, to which a chemical structural formula corresponds.

In chemical terms, a molecule, like individual atoms, has localisable "orbits" (orbitals) of welldifferentiated and isolated electrons. In quantum mechanics, electrons have no individuality because of the Pauli principle: "You cannot label the electrons, you cannot 'paint them red' ", says Schrödinger, "and not only that, you must not even think of them as labelled, otherwise you will get wrong results by 'false counting' at every turn. " (Schrödinger 1962, p. 118) Philosophically speaking, electrons are not substances that can be localised at any time and always remain the same in the change of phenomena. The chemist's electrons ("quasielectrons") must therefore be separated from each other and localised to orbitals in an abstraction step (Primas 1985). In the Hartree-Fock approximation, certain orbitals ("quasi-electrons") are therefore introduced by Schrödinger equations with an external electrostatic potential determined by the classical point charges of the atomic nuclei and the continuous classical charge distributions of the remaining quasi-electrons. There are no quantum mechanical correlations between these quasi-electrons. They are only classically coupled by an electrostatic potential. In this abstraction step, a molecule is thus described as if it consisted of a classical core framework and classical quasi-electrons on localisable orbitals. However, the reality of the molecule with all its entanglements in the quantum world is only captured in the full quantum mechanical description, which, however, cannot provide practical calculations of chemical molecules. In addition to the Hartree-Fock method, other approximation methods should be mentioned that approximate quantum mechanical energy states and state functions of electrons.

Only after these methodological preparations can we turn to the question of which symmetries the spatial molecular structures have with their electron orbitals. Free molecules are not influenced in their geometry by the interaction with neighbouring molecules. One can imagine such a state approximately realised in the gas phase under low pressure. The symmetries of crystals are attributed to molecular lattices. The symmetries of a free molecule can be completely determined by a few types of symmetry transformations. For this purpose, the equilibrium position of the atomic nuclei is assumed. It is often convenient to place the molecule in a Cartesian coordinate system for orientation. As a rule, the origin of the coordinate system is placed in the centre of gravity of the molecule. The symmetry transformations such as mirroring, rotation and inversion result in symmetry transformations when performed one after the other and define the symmetry structure of the molecule through the group of these symmetry elements. Due to the finite number of combinations of the symmetry elements, it is clear that there can only be a finite number of point groups. Therefore, many different molecules can belong to the same point group, i.e. have the same symmetry structure.

The classification of point groups also makes it possible to specify the connection between optical activity and molecular structure mathematically with group theory. According to Pasteur, optical activity of a compound was present when the molecules in question could not be made to coincide with their mirror image. In this case, Pasteur spoke of dissymmetry. Other terms are "enantiometry", which means opposite shape in the Greek translation, or "chirality", which alludes to the left-handedness and right-handedness of the mirrorimage orientation. In terms of group theory, it is important to determine the symmetry elements that lead to optical activity. In general, it is now true that 1) a molecule with any mirror axis cannot be optically active and 2) a molecule without a mirror axis is optically active. Molecules with symmetry groups that do not contain any symmetry element apart from the identity mapping are called asymmetric.

Point groups describe the symmetries of stationary molecules in a state of equilibrium. In the nonstationary case of translational, rotational movements, oscillations, etc., lower symmetries may be present. Scalar properties that have only a magnitude but no direction, such as mass, volume or temperature, are independent of the symmetry operations. However, if one examines properties that not only have a magnitude but also a direction, symmetries can be influenced (Hollas 1975, chapt. 2).

In addition to the symmetries of a molecular nuclear framework, the symmetries of the electron orbitals of a molecule must be determined. Molecular orbitals are often introduced approximately as a linear combination of the atomic orbitals of the individual atoms of the molecule (Linear Combination of Atomic Orbitals = LCAO method). Kekulé's famous ring structure of benzene provides an illustrative example of orbital symmetry. The planar molecule C_6H_6 consists of 6 carbon atoms, which form a regular hexagon and each bind a single H atom. The electron orbitals can be calculated in the so-called Hückel model. They are used to predict chemical reactions.

This by no means exhausts the applications of symmetries in chemistry. In summary, we can state that the historical emergence of stereochemistry was connected with a significant symmetry problem, namely chirality. Quantum chemistry and group theory are the modern foundations of the symmetry considerations of stereochemistry. From heuristic symmetry considerations, initially made only for simplification, experimentally confirmed evidence emerged that suggested symmetric, dissymmetric and asymmetric structures of the molecular world.

7. SYMMETRIES AND SYMMETRY BREAKING IN LIFE

In the world of molecules, biochemistry bridges the gap between chemistry and biology. The basis is provided by macromolecular chemistry, which comes up with new and characteristic symmetry structures. In contrast to low-molecular chemistry, high-molecular or macromolecular chemistry deals with compounds that are composed of very many elements and therefore have a high mass. It was Hermann Staudinger who laid the foundations of macromolecular chemistry in the 1920s. In contrast to earlier assumptions according to which large molecules had to be held together by special forces, Staudinger proposed a theory according to which polymers consist of long chains of molecules that are connected by the forces that are also common in lower molecules. From the point of view of symmetry, polymerisations are nothing more than polyadditions of monomers whose structural formulae form certain stripe ornaments as known in the frieze groups. Examples are the chains of ethylene. With these structural formulae of macromolecules, one is involuntarily reminded of the ornate strip ornaments of mosques, of "structures of quite unusual simplicity, unity and beauty" (Heisenberg 1959, p. 183). Thus Heisenberg speaks about the symmetries of Arabic strip ornaments with a view to the structure of elementary particles. However, the symmetries of nature also become vivid in the structures of polymerisation.

Symmetrical giant molecules of carbon atoms in the shape of footballs are called "fullerenes" after the engineer R. Buckminister Fuller (1895-1983). They have also been used in architecture in the design of halls and arenas. The cluster C_{60} (with included uranium atom) consists of 60 carbon atoms connected in regular 5 and 6 corners. In macromolecular chemistry, giant molecules are now made of thousands of atoms, which present themselves in artfully symmetrical structures. In Fig. 6, symmetrical giant molecules are formed from small molecular units in which atoms join together in regular polyhedra (Mainzer et al. 2013, Fig. 6). This gives the impression that they are constructed from Platonic solids. Plato would be fascinated by the molecular models of modern chemistry, which, because of their complexity, can now only be generated in computer simulations.

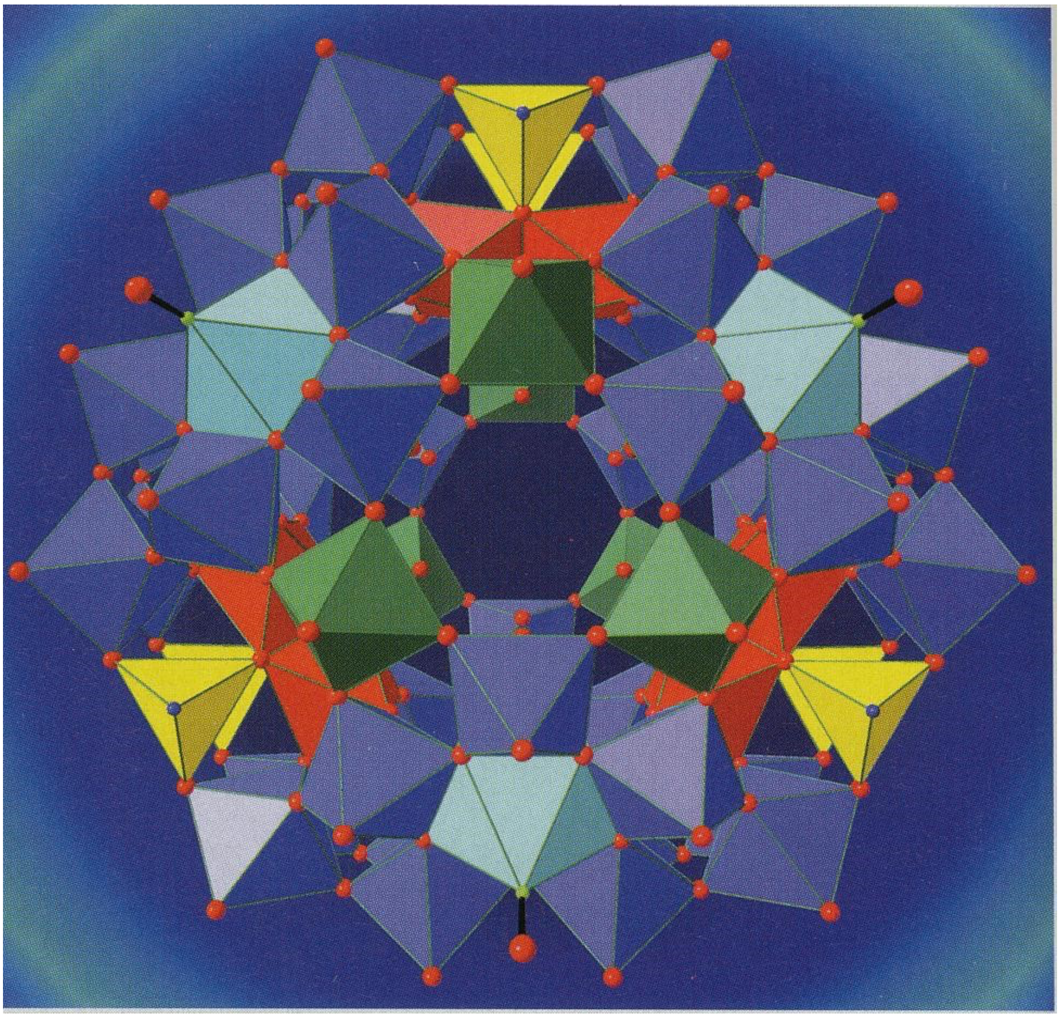


Figure 6. Platonic structures of macromolecules

Some people may also shy away from "cold" symmetry, like Thomas Mann in his novel "Zauberberg" [Magic Mountain], in which he has Hans Castorp say when looking at snow crystals: "Life shuddered at exact correctness." In fact, the large biomolecules as the building blocks of life are characterised by symmetry breaking. Proteins, for example, are made up of many amino acids. Protein analysis reveals that amino acids have an antisymmetric carbon atom and occur only in a left-handed configuration. Fig. 7 shows a lefthanded α-amino acid that changes into a right-handed one by mirroring. If one now examines the spatial configurations of the various amino acid units in the protein, one encounters a characteristic antisymmetry of the protein, in which the antisymmetry of its building blocks is continued.

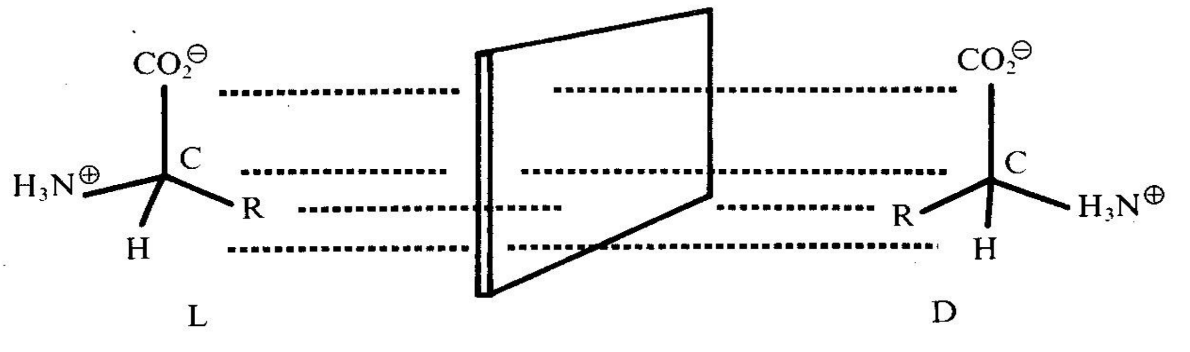
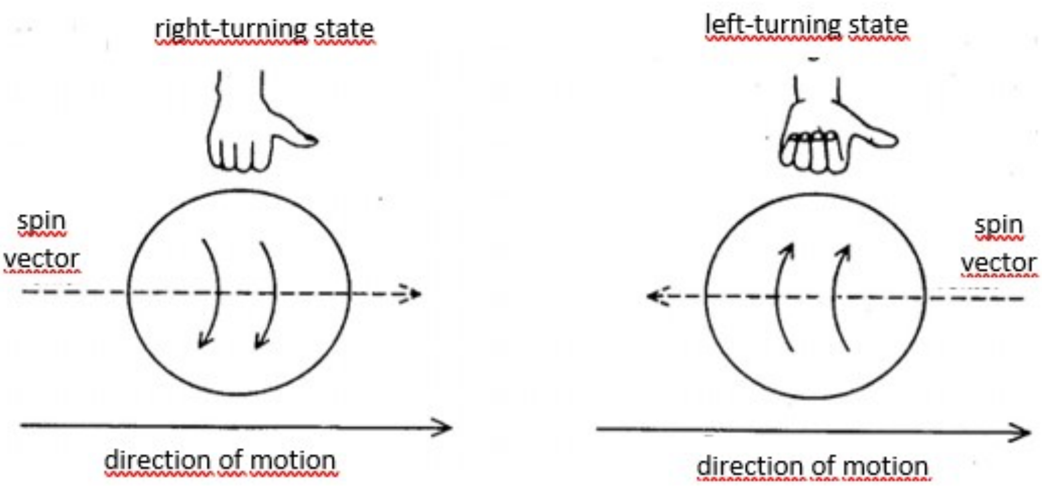


Figure 7. Antisymmetry of amino acids

Nucleic acids, which play a decisive role in the hereditary processes of living organisms, also exhibit characteristic symmetry breaks. Antisymmetry is of fundamental importance for the process of heredity, which can be explained at the molecular level with the DNA spiral. Biochemically, living tissue consists largely of dissymmetrical molecules, of whose two left- or right-handed possibilities usually only one antipode is realised. In particular, the human organism consists of complex dissymmetrical molecular structures. This realisation has practical consequences for pharmacy when it comes to finding out the optimal effect of an antipode in a drug or keeping it away from the human body in any case. Analogous to antimatter on a subatomic level, one could theoretically design a biochemical anti-world in which, for example, people live who are made up of the opposite enantiomers, such as right-handed amino acids and left-handed sugars. In contrast to our biochemical world, in which, for example, right-handed glucose tastes sweet and left-handed glucose does not, the opposite case could exist in the biochemical anti-world, since the taste buds are also dissymmetrical in their molecular structure and would be built up accordingly in reverse. In a certain way, there is even a parallel to the mutual "annihilation" of matter and antimatter, if one thinks of the metabolic disease phenylketonuria, which leads to insanity when a small amount of left-handed phenylanaline is added to human food, while the right-handed antipode does not cause a corresponding catastrophe.

Pasteur even put forward the general hypothesis that the production of single-minded optically active compounds was the prerogative of life, and that through this the only clear dividing line could be drawn between the chemistry of inanimate and animate matter. To this end, he developed a general theory of universal dissymmetric forces that would cause the formation of chiral molecules and crystals. Inspired by Faraday's investigations on magnetically induced optical rotations, he conducted experiments with crystals and magnetic fields. He interpreted the solar system as a dissymmetric rotation and accordingly tried to produce optical activity of synthetic products by running the chemical reactions in a centrifuge. Although Pasteur's physicochemical theory did not apply in detail and the experiments remained negative, it is an interesting attempt to attribute dissymmetry in biochemistry to physical causes.

For some years now, ab initio calculations of quantum chemistry have been available with which the observed chirality of biomolecules could be attributed to a physical symmetry breaking. This refers to the parity violation of the weak interaction that occurred with cosmic symmetry breaking when the weak interaction was separated. In the weak interaction, there are only left-turning neutrinos, but no right-turning neutrinos (parity violation), as illustrated in Fig. 8:



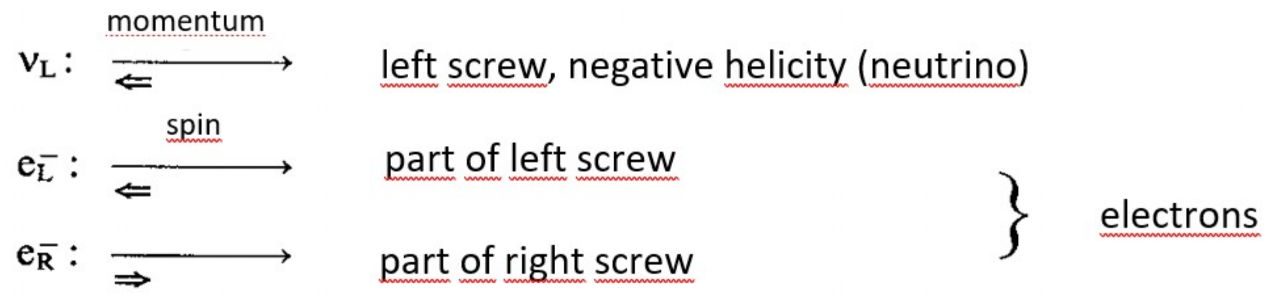


Figure 8. Symmetry breaking of the weak interaction

Methodologically, this causal explanation would be interesting because it traces homochiral biochemistry back to racemic chemistry, which in turn could be traced back to the quantum field theory of the fundamental physical forces in the context of quantum chemistry. In chemistry, the same energy is usually assigned to the right-handed and left-handed copies of a chiral molecule. According to this explanation, the parity violation of the weak interaction in a molecule contributes a tiny amount to the electronic binding energy. Although this contribution is the same in the left-handed and right-handed copies of the molecule, it carries a different sign + or - because of the parity violation of the weak interaction. The energy of one copy is therefore increased a little by this amount, the other copy is decreased by the same amount. The difference between the two energies is called the parity-violating energy difference ΔEpv (Fig. 9) which is based on ab initio procedures of numerical quantum chemistry (Tranter 1986, p. 866; Quack 1986). Thus, it is a hypothesis which still needs confirmation by measurement.

From a methodological point of view, the example of parity-violating energy in quantum chemistry underlines the necessity of empirical measurement to confirm or to refute hypotheses on symmetry and symmetry breaking in natural sciences. The parity-violating energy results in the chiral copy with the low energy being stabilised by a tiny amount relative to the other chiral copy. Therefore, in a racemic mixture in which the left-handed and right-handed copies occur in the same way, a tiny excess of those chiral copies with the lower energy in each case will be found. This stabilisation can also be assumed for the transition states in chemical reactions, in that the parity-violating energy difference leads to differences in the reaction rates and thus to a preference for certain chiral copies. The hypothesis is that these small differences could cause the observed homochiral selection of biomolecules:

The Δ Epv values for some amino acids were calculated using the Hartree method, i.e. a very precise quantum chemical method. This clearly predicted a lower energy of the left-handed specimens compared to the right-handed ones. The left-handed amino acids form the building blocks of proteins in polymeric form. For the polymerisation of the left-handed amino acids in an α-helix, a significant increase in the Δ Epv value over a single amino acid can be demonstrated when the individual Δ Epv values of the individual amino acids are increased in the sense of polyaddition. The polymerisation of macromolecular chemistry thus acts here as a chiral selection enhancer.

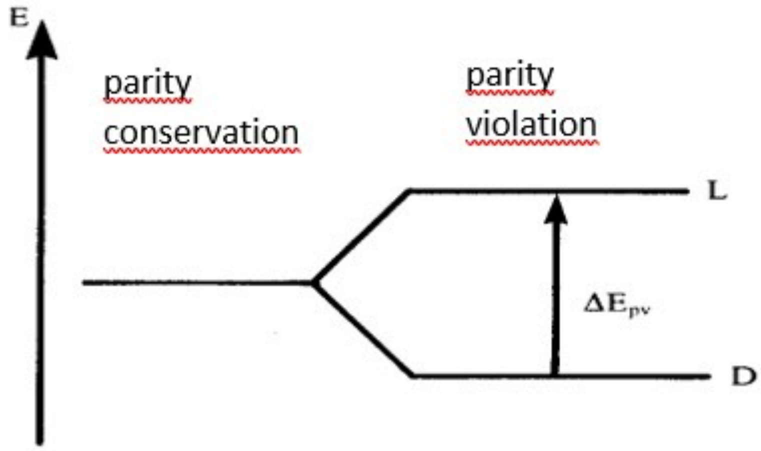


Figure 9: Chemical symmetry breaking through through parity-violating energy?

The individual energy differences of single molecules are certainly very small. Even if these differences increase proportionally during polymerisation, they still remain very small under laboratory conditions. In evolution, however, nature itself was the laboratory. For amino acids, for example, it is possible to calculate exactly the prebiotic evolutionary conditions under which homochirality can be selected, e.g. in a lake with a certain water volume and in a certain time. Finally, it should be emphasised once again that these calculations are based on ab initio procedures of numerical quantum chemistry. However, this would require experimental confirmation of measurements of parity-violating energy.

Biochemistry and molecular biology are the foundations of modern biology and therefore serve as the basis for explaining the life processes of organisms such as bacteria, plants and animals. Nevertheless, organisms are not simply complex aggregates of atoms and molecules. Some symmetry properties are indeed determined by building blocks (e.g. dissymmetry of proteins). However, at the higher level of organisation of organisms, new properties of symmetry, dissymmetry and asymmetry emerge that are necessitated by functional requirements (e.g. adaptation to the environment, species preservation, metabolism) (Mainzer 1988, chapt. 4.43).

If we take a scale of aggregates between the highest symmetry and chaos as a basis, from e.g. crystals to para-crystals, liquid crystals, gels, real liquids to ideal liquids and gases, then living organisms are classified in the middle range. Their symmetries are of a statistical nature, since invariant patterns of properties recur during self-reproduction, but also more or less random changes occur and thus only diversity and individuality of living nature become possible. The sudden changes in the genetic material (mutations) are typical symmetry breaking processes of evolution. Often unicellular organisms, such as the unicellular green algae, have a main axis. Vegetative reproduction then usually takes place by longitudinal and oblique division of the cell. In other bacteria and algae, depending on the species, regular division takes place in one, two or all three spatial directions, which can lead to different cell assemblies, e.g. as bead-like chains such as streptococci, filaments such as the filament-forming joch algae or colonies with division of labour of the individual cells.

In higher plants, symmetry elements such as translation, rotation and mirroring can be distinguished. Rarely is there pure translational symmetry, in which the leaves grow isometrically along the shoot axis with the same translational length. More often, growth is associated with a lawful dilation of the translational units, such as the leaflets of the mountain ash. This example exhibits lateral symmetry in addition to longitudinal symmetry, as the left and right leaf halves are mirror images of each other. In contrast, the elm branch is produced by a gliding mirroring, in which a translation and a mirroring form a combined symmetry operation.

Screw movements also occur in leaf positions, where the translation of the growth movement is associated with a unit of rotation. An example is the helically leafed shoot of a species of fat-hen, where each leaf has a definite angle of divergence with the preceding one. The positions taken by a moving point at evenly spaced times in a time interval are evenly distributed over this helical line - analogous to the steps of a spiral staircase. It turns out that the fractions μ/ν used to represent the helical arrangements of leaves are often members of the Fibonacci sequence: 1/1, 1/2, 2/3, 3/5, 5/8, 8/13, ... This sequence results from the continued fraction of the irrational number $(\sqrt{5} - 1)/2$, i.e. the proportion ratio of the golden section. If you replace the cylinder on which the helical line moves with a cone, you get a symmetry operation in which, in addition to translation and rotation, dilation also occurs. An example is the arrangement of the scales in a pine cone. In the inflorescence of a giant sunflower (Helianthus maximus), the small flowers arrange themselves in logarithmic spirals, with two sets of spirals with opposite sense of winding.

As a rule, the shape symmetries of animals are based on functional requirements that are intended to ensure sufficient fitness for life. One of the most common symmetries in animal organisms is mirror symmetry. More than 95% of all animal species belong to the bilateralia with simple mirror symmetry. Locomotion in a direction transverse to gravity is characteristic of the crawling, running, swimming and flying Bilateralia such as snakes, lizards, fish, insects and birds.

Since ancient times, there has been speculation as to whether the bodies of humans and animals are constructed according to a certain canon of proportions (e.g. the golden section). As examples, the proportions of a butterfly (a) and a fish (c) are shown in Fig. 10. The ratio of the golden section is determined by the flow conditions of the respective medium (air and water) in which flight movements or flight-like swimming movements (as in the case of a ray) are carried out. In evolutionary theory, the Platonic assumption of an a priori predetermined art form of nature is replaced by the selection advantage that distinguishes one particular symmetry from another. Rotational symmetry, for example, has a selection advantage for free-floating or sessile marine animals such as medusae and flower animals, which Ernst Haeckel already traced meticulously as "art forms of nature".

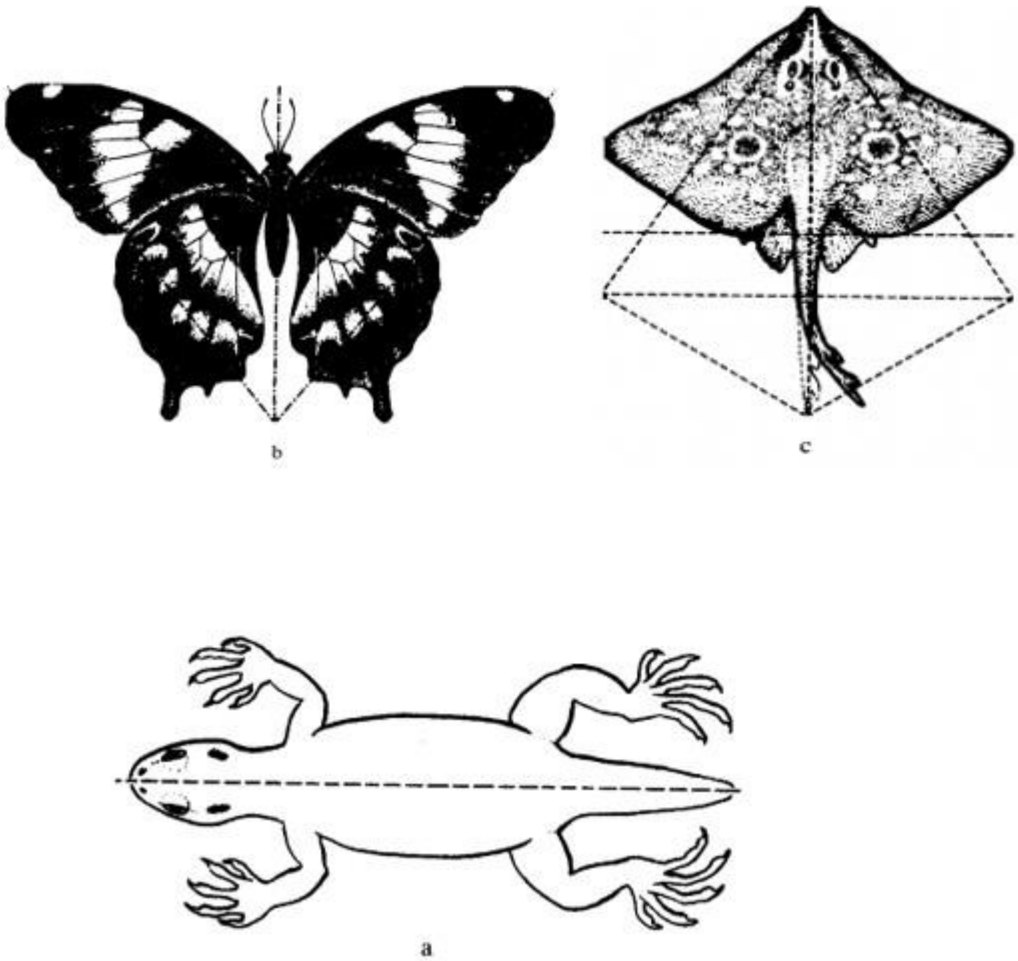


Figure 10. Bilateral symmetries of flying, swimming and running animals

The bilateral symmetry of the body structure also continues in the structure of the nervous systems up to the evolution of the brains. In insects and worms, the development of a mirror-symmetrical rope ladder nervous system can be traced, which in the course of evolution led to concentrations in the head and finally to highly developed brains like those of humans. Fig. 11a shows the human central nervous system with brain, spinal cord and spiral nerves. Fig. 11b lays out a horizontal section through the human brain (Warwick and Williams, p. 35). The phylogenetically older structures in the centre are completely mirror symmetrical. However, the asymmetry of the cerebral cortex (cortex cerebri) is obvious. It is caused by new complex functions of information processing in later times.

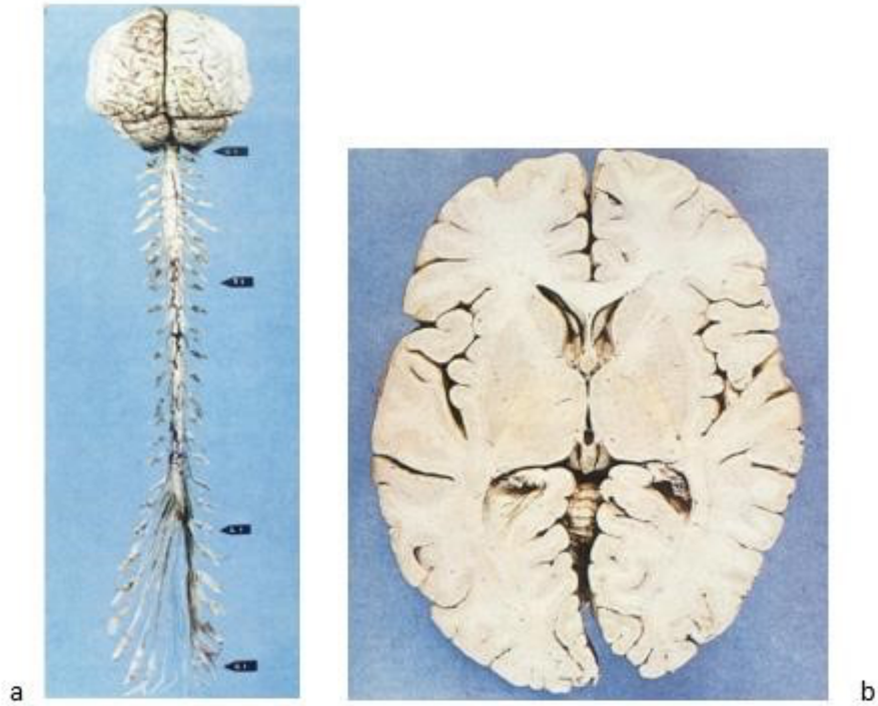


Figure 11. Symmetry and asymmetry of the brain anatomy

Symmetry and symmetry breaking are not only principles that determine the emergence of patterns and structures in nature. They also enable pattern and structure recognition, as can be illustrated in Gestalt psychology using the example of tilt pictures. In Fig. 12a, do we see a symmetrical vase or two diametrical faces? Those who first think they see a nose tip, for example, will spontaneously recognise two diametrical faces in which the white surface forms the background. But whoever spontaneously recognises a white handle in the diametrical edges will see a chalice on a black background in the foreground. The ambiguity of the perception of such "tilt pictures" and the spontaneous decision of the cognitive system for an interpretation is a psychological example of spontaneous symmetry breaking that depends on the smallest details of individual perspectives.

Mathematically, spontaneous symmetry breaking in designer cognition functions analogously to pattern formation in nature. Spontaneous symmetry breaking means that a state (e.g. position of a sphere) at the maximum of a potential (e.g. peak of a curve) is symmetrical but unstable (Fig. 12.a). Tiny initial fluctuations then decide which of the two possible minimal states is assumed by the system (e.g. sphere) (Fig. 12b). Spontaneous symmetry breaking in the emergence of the basic physical forces of nature during cosmic expansion (Fig. 2) also works according to this principle, albeit under the conditions of quantum physics.

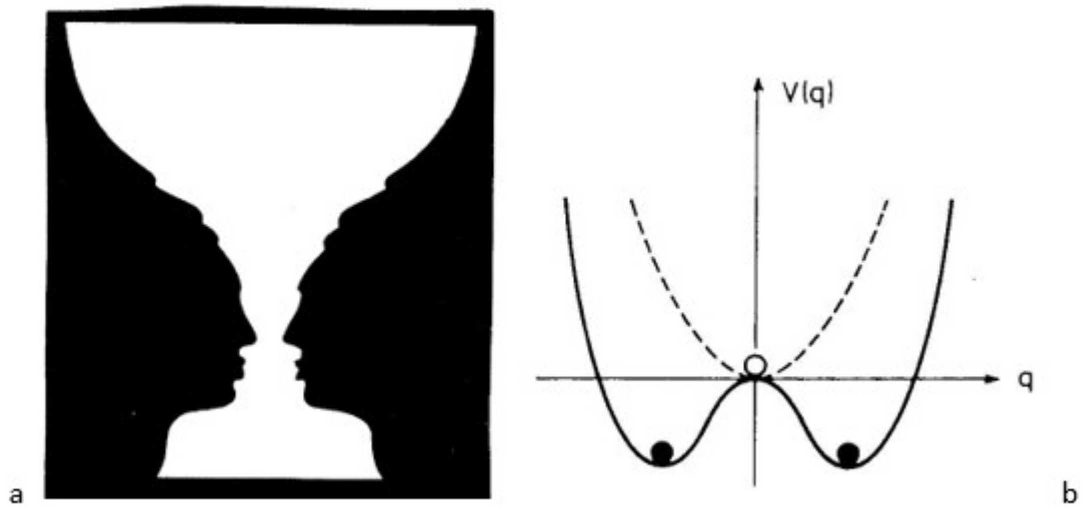


Figure 12. Spontaneous symmetry breaking of Gestalt cognition.

8. SYMMETRY AND SYMMETRY BREAKING IN MODERN MUSIC

The Pythagorian conception included music in the quadrivium of the exact sciences as part of their doctrine of harmony along with geometry, arithmetic and astronomy. For the Pythagoreans musical harmonies had the character of natural laws, that is, they were expressions of symmetry laws of nature such as the harmony of the spheres in astronomy. In other early cultures, musical harmony, nature, and life are also identified with each another. In the occidental tradition this unity was broken by the end of the Renaissance at the latest. Aesthetic interpretations and research in the natural sciences developed their own unmistakeably distinct categories and laws. Antique standards of art were explicitly criticized by later periods, and their ontological grounding, as in the Pythagorean tradition, was called into question.

On the other hand, we have noticed that representational art and mathematics have areas that overlap. In music one immediately thinks of the Baroque, of course, and especially of Bach with his elaborate fugues (Werker 1922). However, as long as the concept of symmetry is limited to antique ideas of proportion, or even – as occurs frequently in modern theory of the arts – to reflection symmetry – symmetries in music must seem more or less accidental and sporadic. It was the new revolutionary breakthroughs in music in the 20th century such as 12-tone music, that made the connections with the encompassing mathematical concept of symmetry clear.

From a mathematical point of view, it is apparent that fundamental concepts of music theory can be translated into the group-theoretical language of modern mathematical symmetry theory (Claus 1980, pp. 70, 76). That makes it possible to analyze examples of music from the Medieval modes through Bach, Beethoven, Schönberg, et al. This common language of mathematical, scientific and artistic subjects not only fosters the unity of the „two cultures", a unity which was thought to be lost.

Yet this new unity of methods in mathematics, art and natural science is sustained by fundamentally different intentions than those that obtained for the Pythagorean quadrivium. It is no longer possible to consider tone scales and harmonies to be the expression of particular natural laws. What we are looking at now is a unity of methods, not an ontologically based unity like that of the Pythagoreans. The new methods facilitate intercultural comparison for working out the structures held in common and the differences.

In the beginning of the 20th century, it was A. Schönberg who broke with the classical canon of harmony totally in order to allow for greater potentailities of form in the 12-tone technique. The symmetries of tone scales and harmonics can be described as cyclical mathematical groups and thus traced back to rotation symmetries. Thus, in Fig. 13 (1), the 12-step half-tone scale constitutes a cyclic group C_{12} with the half tones C, Cis, D, Dis, E, … et al. geometrically illustrated as rotations of a 12-polygon with rotations of angles $0 \cdot \pi/6$, cyclical subgroups for the three types of diminished seventh chords (Fig. 13 (3)), whole rotations of $n \cdot 2\pi/3$ analogously produce four cyclical subgroups for the four different triads (Fig. 13 (4)).

It is remarkable that clockwise rotations of $n \cdot 5\pi/6$ from C transform the semitone circle into the circle of fourths, and counterclockwise into the circle of fifths. In harmonics, triads can be represented by (nonequilateral) triangles in a semitone circle. Major and minor triads form congruent, but mirror-symmetrical, triangles (Fig. 13 (5)). Further studies of cyclic groups in harmonics can be carried over the other tone scales and permit intercultural comparisons of different conceptions of harmony.

Symmetry operations such as translation, reflection or rotation can be examined in a musical space which is defined by the dimensions of time, frequency, and dynamics. Thus translations in the dimension of time are interpreted as repetition of tones; translations in the dimension of frequency are interpreted as the parallel direction of voices; translations in dynamics are interpreted as crescendo or diminuendo. Rotations such as C_2 can be examined on the axis of dynamics, that is on the notational plane of time and frequency. Reflections in the plane of time and frequency correspond to an increase or decrease of the volume. Reflections in the plane of dynamics and time are possible, that is, on the horizontal plane of the notation. Reflection along the vertical, that is, the plane of frequency-dynamics, are more familiar.This is the so-called retrogression which appears frequently in the 15th and 16th century and in classical examples by Bach and is systematically in the serial music of Schönberg's 12-tone technique.

The question as to whether the combination of symmetry operations is applicable to musical ornaments, marks a highlight in this kin of group-theoretical analysis. Examples of notation in the work of Bach can be systematically cited for the seven one-sided stripe ornaments in the notational plane. An example of a sequence of ornament symmetries is measures 27-29 of the first movement of the piano sonata, op. 53 („Waldstein Sonata") of L. v. Beethoven. What comes into consideration are the stripe ornaments of the frieze groups.

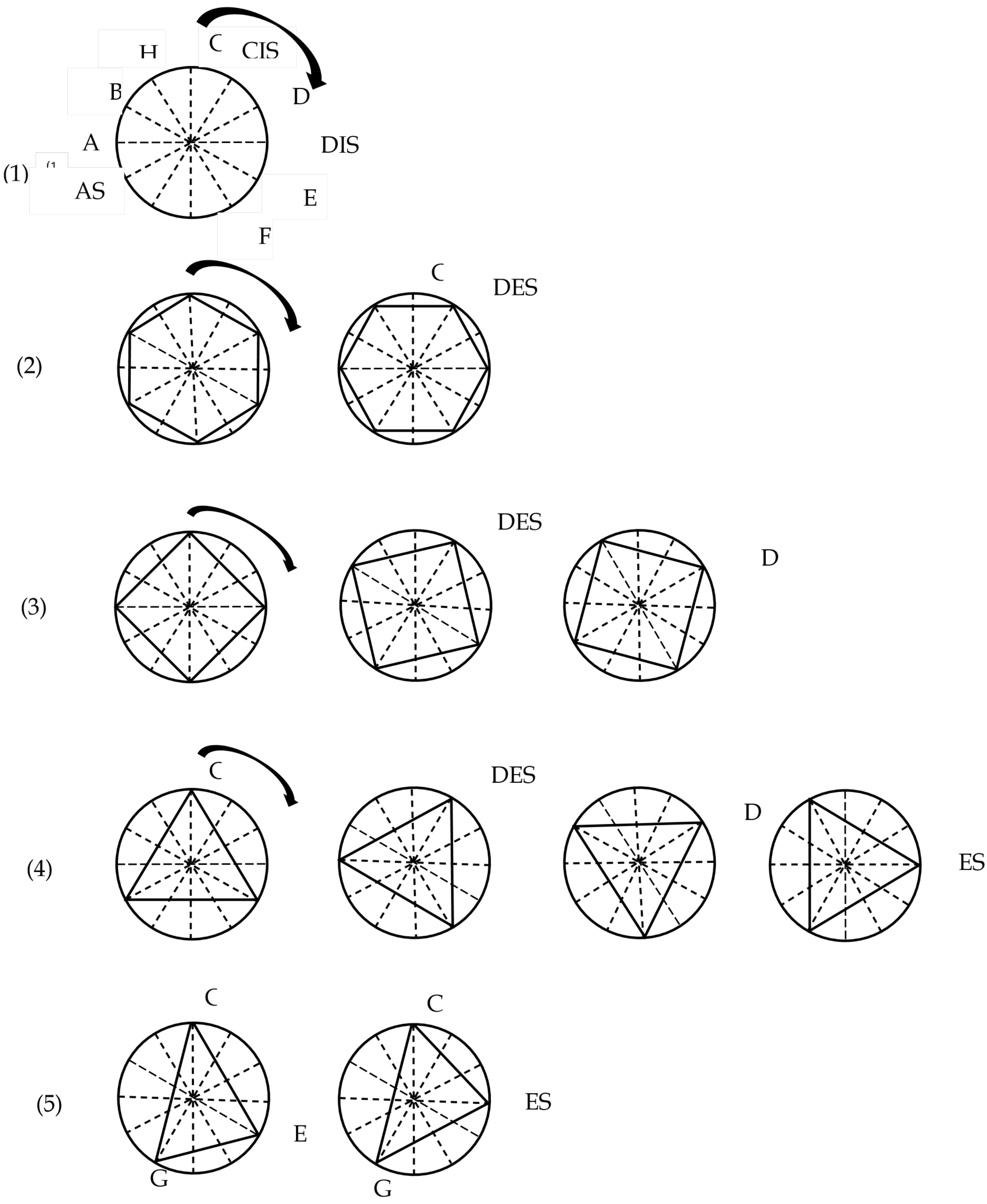


Figure 13. Mathematical symmetry groups of 12-tone music

9. SYMMETRY AND SYMMETRY BREAKING IN MODERN PAINTING AND ARCHITECTURE

In modern times the common basis of science and art broke apart. Mathematical natural science developed an abstract concept of symmetry that went beyond the geometrical theory of proportions. Art was no longer oriented to a strict geometrical canon of proportions according to the Antique model. Yet geometrical views of symmetry played a great role in certain epochs such as the Baroque. And - though not in the sense of quantitative regularities – Classicism and Romanticism spoke of „inner perfection" and „harmony of the soul". According to Schelling, the cosmic harmony was expressed in them.

Thus, the concepts of symmetry and harmony were further developed independently in sciences and art. In the beginning of the 20 th century modern art's breach with tradition coincided strikely with the paradigm shift that scientific theory underwent for the development of modern physics. The concept of symmetry took on a preeminent role in this new development of mathematical natural science. My thesis is that modern art is seeking new common structural laws (Mainzer 1988, chapt. 5.4).

The fundamental revolution in art took place during the first decade of the 20th century in the rise of abstract art. Its goal was to work out a method of representation that would allow the painter to express his view of the world without having to draw on graphic objects and their superficial appearance. There is an astounding parallel to the problems of abstract quantum mechanical formalism, which was to be developed out of graphic classical mechanics. Artists like Picasso and Braque certainly did not read Planck, Einstein or other physicists. Their cubism had instead followed Cézanne's pronouncement that objects are made of geometrical forms such as spheres, cones, and cylinders. In addition there is the recourse to archaic art. The objects represented in the cubist pictures are broken up into stereometric atoms and then reassembled in a new way for the purpose of making the basic and archetyal forms of the world vivid. In 1912 a theory of cubism was formulated.

Artists like Paul Klee applied the ideas of symmetry and law to abstract art. He particularly pointed out the parallel with mathematical natural science. In his study „Exact Experiments in the Realm of Art", he wrote:

„Art has also given enough space for exact research, and the gates have been open to it for some time. What was done for music before, until the end of the 18th century, is at least beginning in the field of sculpture. Mathematics and physics are providing the opportunity, in the form of rules for persistence and for alteration.

This compulsion to concern oneself first with the functions instead of beginning with the finished form, is a wholesome one. Algebraic, geometric and mechanical tasks provide training toward the essential, the functional, in contrast to the impresssive. One learns to see behind the facades and to grasp a thing by its roots. One learns to recognize what is flowing underneath it – the prehistory of the visible – and to dig into the depths and to expose, substantiate and analyze." (Klee 1928)

While modern painting has remained limited to pictorial representation, in the twenties of the 20th century the Bauhaus set about giving artistic form to the technical-industrial life world a a synthesis of the arts of architecture, painting, sculpture and functional art. Here as in Antiquity it was a matter of comprehending the human being and his life world as a unity, but now it was from the point of view of science, technology, and industry.

Along with the Bauhaus, there were a series of other representatives of modern architecture that could be pointed out. One was the Dutch group represented in the magazine „De Stijl". Theo van Doesburg and Cornelis Eesteren emerged as leading theorists of this group. In their article „On the Way to a Collective Construction" (1924), they called for structuring the life world „according to creative laws" that emerge from a „creative principle" (van Doesburg and van Eesteren 1924). They went on to emphasize: „These laws, which are linked to the economic, mathematical, technical, hygienic laws, lead to a new sculptural unity".

Erich Mendelson, widely known as the builder of the Einstein Tower in Berlin, 1923 asserted the logocentrism of modernism: „Seldom – it seems to me – has the order of the world been revealed so distinctly, only seldom has the Logos of being opened wider than in this time of supposed chaos." This pronouncement has an effect like Heraclitus' reference to the „hidden harmony" in chaos. One of the central representatives of modernism was Le Corbusier, who not only achieved magnificent edifices, but also came onto the scence as a theorist. A passage from his manifesto on „Urbanism" (1925) follows which elevates Le Corbusier outright to a Platonist of modern architecture:

„Geometry is the means we ourselves have created for ourselves so that we can overcome our surroundings and express ourselves. Geometry is the foundation. It is at the same time the material bearer of the symbols that signify perfection and the divine. It bestows on us the sublime satisfaction of mathematics. The machine proceeds from geometry; its dreams set out to find the joys of geometry. The modern arts and modern thinking, after a century of analysis, seek their salvation beyond accidental facts, and geometry conducts them to a mathematical order, … ." (Le Corbusier 1925)

One could almost speak of a „logos of modernism" in which symmetry and functional unity would occur as structural characteristics. Art, architecture, and science would then be only different forms of expression and projections of one and the same temporal epoch. For philosophy, analogously, one could name Wittgenstein's „Tractatus Logico-Philosophicus", Carnap's „Logical Structure of the world" or Neurath's „Unified Science", which express the logocentrism of the era.

Corresponding to the paradigm shift in science there was thus an artistic upheaval and structural change striving toward a new measure of structures. The standard is a purpose-oriented functionalism that aims to comprehend the human being in his new life world. The words „new" and „modern" became the fashion in the twenties of the last century, which saw the political and social collapse of old world orders. The use of the word „new" ranged from the „Neues Bauen" (new constructions) and „Neues Wohnen" (new dwelling) by the way of the the „Neue Sachlichkeit" (new practicality) to Huxley's „brave new orld", with its disillusionment and irony. Interrupted by the 2nd World War, many projects of modernism were not actualized of further development until the fifties and sixties of the last century.

The doubt and disenchantment that set in about the modern industrial culture began to be discussed in architecture under the catchword of postmodernism (Jencks 1978). This signalled breaks of symmetry in the functionalistic structure of the modern life world, including breaks of style in architecture, deviations from industrial funtionalism, disturbances in the ecological balance, and alternative life forms. Postmodernism raised a question of whether what had occurred was really a „loss of the center" and a loss of ideas and orientation, or whether it was instead an expression of the complementarity of incompatible ways of seeing that are ultimately related to a „hidden harmony" of a chaotic world in Heraclitus' sense.

Here it is important for modernism to emphasize again that its „center" and „symmetry" are not to be confused with external simple symmetry characteristics such as reflection symmetry or axial symmetry. In modern natural science as well, the external geometrical symmetry characteristics of individual bodies emphasized in Antiquity, play a rather subordinate role. What is decisive are the uniform, comprehensive (but abstract) symmetry characteristics that are expressed in the mathematical structures of science. Analogously, the concept of symmetry that is intended in the architecture of modernism should be seen as a characteristic of a uniform structuralism and functionalism. In this sense the architecture of postmodernism comes to „breaks of symmetry".

10. PHILOSOPHICAL OUTLOOK ON SYMMETRY AND SYMMETRY BREAKING

Mathematical symmetries and symmetry breaking can thus be demonstrated in physics, chemistry, biology, brain research and psychology as well as in arts. The epistemological question that has been asked since antiquity is: Are symmetries only constructions of the human mind or real structures of the world? Are symmetries only properties of scientific theories and models produced by human brains to reduce complexity or following aesthetic purposes? But if they are only mathematical constructions, why do observations, measurements and predictions provide these regularities?

On the other hand, fundamental symmetry breaking takes place in nature. As shown in the second section, the basic physical forces that split off from the unified primordial force during the cosmic expansion are characterised by partial symmetries. With each of these partial forces, a multitude of new elementary particles emerge, which eventually combine to form atoms, molecules, gases and materials. The symmetry and simplicity of the primordial beginning is lost in the process and splits into cosmic diversity and complexity. In the evolution of life, the unfolding of complexity continues through symmetry breaking.

Symmetry and symmetry breaking are thus complementarily related to each other. Symmetries of theories open up insights into invariant basic structures of the world. Symmetry breaking enables becoming and opens up insights into the diversity and complexity of the world.

Since Plato, the view has been expressed that the mathematics of the laws of nature is a sign of the symmetry of the universe, pointing to a central order. For Stephen Weinberg, also a Nobel laureate like Heisenberg, Dirac and Wigner, who have studied the mathematical symmetries of modern physics, the cosmic world and its laws tend to be implacable, forbidding and hostile to human beings. What remains is the common insight into fundamental basic structures of the world: conservation laws, natural constants, and symmetries.

According to the Platonic view, we humans can "participate" in the world of mathematical structures that underlie the physical world through our logical-mathematical cognitive ability. This would be the ultimate explanation of why mathematics fits so well with the physical world: The fundamental reality is mathematics itself and we live in one of the mathematically possible universes. Heisenberg held this Platonic view: "The ultimate root of phenomena is therefore not matter, but mathematical law, symmetry, mathematical form." (Heisenberg 1959, p. 163) According to Plato, the true is at the same time the beautiful and the good. We will not go that far today. However, it is undisputed that the mathematical universes comprise far more structures than have been discovered so far in the physical world. We have to select the appropriate mathematical structures through measurement procedures, experiments and observational data (Mainzer 2014, chapt. 14). Symmetries remain a regulative guiding idea of research, as they have been since antiquity.

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