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On the incapability of inertial forces as a means of repeated self-propulsion of an object in a vacuum

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ABSTRACT

This paper deals with the controversial topic of "inertial propulsion". It discusses in depth the limits of possible upward vertical motion of a vehicle equipped with a couple of contra-rotating masses. Although inertial forces are internal forces in the system, they can cause the center of mass to move upwards up to a certain distance (first cycle of motion), provided the vehicle is initially supported, for example on the ground surface. To make the matter easier to understand, we also contrast it with the operation of the mass-spring system. In contrast to many patents which claim that the key ingredient to achieve a supposed "net thrust" is to invent a way to achieve nonzero impulse of the inertial forces in each rotation of the masses, this paper shows that the main reason of the motion is the initial velocity of the center of mass which is associated with the initial orientation of the rotating arms carrying the masses. Moreover, it is shown that the pattern of the angular velocity only affects the vehicle's velocity while the position of the vehicle slightly oscillates around the standard position of the center of mass which always performs the motion of a mass particle in vertical shot. In the end, it was shown that non-zero thrust of the inertial forces per revolution is possible in many ways, but in no case ensures repeated propulsion of the vehicle in a vacuum, the realization of which requires either intermediate support conditions or reaction forces at the time when the maximum travel in each movement cycle is completed.

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Introduction

“Inertial propulsion” refers to a concept in physics and engineering where a system or device supposedly generates thrust without ejecting mass, often by using internal mechanisms to manipulate or redistribute its own mass. The idea is to create motion by changing the distribution of mass within a system, thereby generating a net force that propels the system forward. This concept challenges the traditional understanding of physics, particularly Newton's Third Law of Motion, which states that for every action, there is an equal and opposite reaction (Provatidis, 2024). The concept of inertial propulsion has been developed out of a desire to create new, efficient means of propulsion that do not rely on conventional methods, such as chemical rockets that expel mass (e.g., fuel) to generate thrust. One of the main reasons why this concept has attracted interest is that it would cause efficient space travel. In more details, in traditional space travel, spacecraft must carry large amounts of fuel to generate thrust, which limits their payload capacity and range. If inertial propulsion were possible, it could potentially allow spacecraft to travel great distances without the need for large fuel reserves, making long-duration missions, such as interstellar travel, more feasible (Robertson *et al*., 2008; Robertson and Webb, 2011).

The origin of this idea goes thousands of years back to the Ancient Greece, where athletes used halteres to elongate their long jump in the Olympic Games (Minetti and Ardigó, 2002). Modern history is about a century old. It started from many kinds of people worldwide such as professors of engineering, practical engineers and others. At the risk of omitting some of them, typical countries are Italy (Todeschini, 1933; Di Bella, 1967) and Russia (Tolchin, 1977; referring to activities since early 1930s), while the most striking appearance was that of the so-called *Dean drive* (Dean, 1959) in the USA. In 1970s the idea was extended to gyroscopes by the English Professor Eric Laithwaite (1970, 1974), the "Father of Maglev". Later, in the turn of the 21st century, several people from academia conducted research in similar concepts: in the USA (Almesallmy, 2006), Bulgaria (Loukanov, 2014), China (Wang and Ju, 2014), Romania (Timofte *et al*., 2023) among others, either theoretically or/and by manufacturing prototypes. The cybernetics of inertial propulsion for a reflecting object in impact have been studied by Engel and Stiebitz (2009). During a Conference in 2015, a principal engineer of the Boeing Company (USA) announced that since 1960s they were conducting research on the amount of induced inertial forces in the huge control moment gyroscopes (CMGs) the company used in the space, and eventually he claimed *net thrust* (Gamble, 2015). In 2013, Provatidis and Gamble (2013) tried to erect the reaction forces induced in a rotating elastic bar under Heaviside pressure pinned at one end, but this attempt failed due to the Coriolis force. The full story is very extensive, most of which is covered by a recent comprehensive review of 216 papers (Provatidis, 2024).

In our days, the most familiar case of inertial propulsion in daily life is probably the shake of a cell phone on a table when it rings, due to the bumper (vibrator) in which the inertial forces exceed the static friction, and thus motion is produced (stick-slip phenomenon). Moreover, in the industrial domain, in the last several decades has vibration been applied to obtaining useful effects, and have the so-called vibratory machines and devices appeared. Typical examples are vibro-compactors used in modern hydrotechnical construction, apparatus for concrete mixtures, vibro-machines and devices such as sizing screens, conveyors, feeders, mills, breakers, flotation machines, shakers and many other devices widely used in various industries (Blekhman, 1988; Pfeiffer, 2016). Some other peculiar phenomena related to the lift of objects (e.g. Indian rope trick, *etc*.), which however can be explained in term of structural dynamics, have been reported by Pfeiffer and Mayet (2017).

While all the machinery of the above paragraph are related to terrestrial applications in which *static friction* plays the most important role, a lot of patents have been filed worldwide and some of them granted, with a goal the supposed development of propellant-less spacecrafts for future needs of the mankind. Probably because no patents are allowed when physics laws are violated, some of them are labelled as “educational stuff" aiming at teaching what inertial propulsion is (Pittman, 2019). Note that this kind of propulsion in the space would require energy consumption, but this is no problem because energy can be easily harvested from the environment, for example from sun radiation. The true technical problem is *how to convert the energy into propulsive force without expelling material* and without using advanced external means such as solar sails, *etc*.

While NASA has published a specific technical report to discourage inventors working in this field (Millis and Thomas, 2006; Millis and Davis, 2009), there are still several patents issued and publications in favor of this issue. The utmost purpose of relevant inventors is to manage (control) rotating masses move in such a way that the produced impulse of the inertial forces becomes different than zero (Gutsche, 2018a). Moreover, in out-of-the-stream publications, it has been claimed that when the out-of-balance masses are located at the outcome of a three-bar mechanism, net thrust has been measured (Gutsche, 2018b) and further this has sparked extensive mathematical analyses (Allen, 2019; Allen and Dunning-Davies, 2023). A thorough collection of similar thoughts has been presented by Childress (2010).

Therefore, not only must a convincing and unequivocal fair answer be given to the out-of-stream researchers, but there are still several issues that have not been addressed so far. For example, Hampton (2022) claims that the concept of Dean drive must be slightly modified so as the joints at which the out-of-balance masses are articulated be movable. Moreover, Wang and Ju (2014) claim that they noticed only an oscillation of the system when the friction was negligible. These and many other comments reveal that, despite the research conducted so far, some of the basics in inertial propulsion are still unclear.

In this context, the purpose of this paper is to elucidate several views of inertial propulsion. It will eventually be shown that although the development of non-zero impulse of inertial forces per rotation in a typical Dean drive is *possible*, this factoid does not mean that this net thrust can overcome the gravitational forces of the system. Therefore, inertial propulsion is not a viable solution for alternative propulsion in a vacuum.

Dynamics of contra-rotating masses

In this section we present the basic theory of inertial drives.

Inertial force in a system of masses

In general, according to Newton's Second Law in mechanics, the sum of the external forces $\vec{F}_{ext}$ applied to a system of masses equals the total mass m_{total} times the acceleration $\vec{a}_{cm}$ of the center of mass:

$$\vec{F}_{ext} = m_{total}\vec{a}_{cm} \tag{1}$$

For a set of particles, the proof of Eq. (1) is a common place in physics and is based on two issues: i) the separate use of Newton's second law on each mass of the system; and ii) the assumption of Newton's Third Law (action = reaction). The reader may refer, for example, to a standard textbook of physics by Halliday and Resnick (1966) as well as to monographs by Casey (1994, 2014) and Pinheiro (2010).

Usually, we shift the right-hand side of Eq. (1) into the left-hand side by changing its sign, and thus we receive the equivalent form:

$$\vec{F}_{ext} + \left(-m_{total}\vec{a}_{cm}\right) = 0\,. \tag{2}$$

Therefore, the fictitious "inertial force" (d'Alembert force):

$$\vec{F}_{inertial} = -\left(m_{total}\vec{a}_{cm}\right), \tag{3}$$

can be considered as if it were an actual force, acting at the center of the mass, which is in *static equilibrium* and thus cancels the actual external force $\vec{F}_{ext}$.

Obviously, in case of a single mass particle, Eqs. (1) to (3) are again valid.

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Inertial drive and reaction force

An *inertial drive* is a set of contra-rotating masses which is put on a vehicle (sometimes also called as cart, chassis or object). Without loss of generality, we consider the inertial drive shown in Figure 1. It consists of two massless rigid arms (J_1A_1 and J_2A_2) of equal length r which are attached to the vehicle at the joints J_1 and J_2, respectively. The masses at the endpoints A_1 and A_2 of these arms, each of magnitude m, contra-rotate at the same angular velocity $\omega(t)$, and thus A_1 and A_2 are symmetric with respect to the y-axis. Apart from the dead weights, no other external force (e.g., aerodynamic, friction, etc.) is exerted on the system.

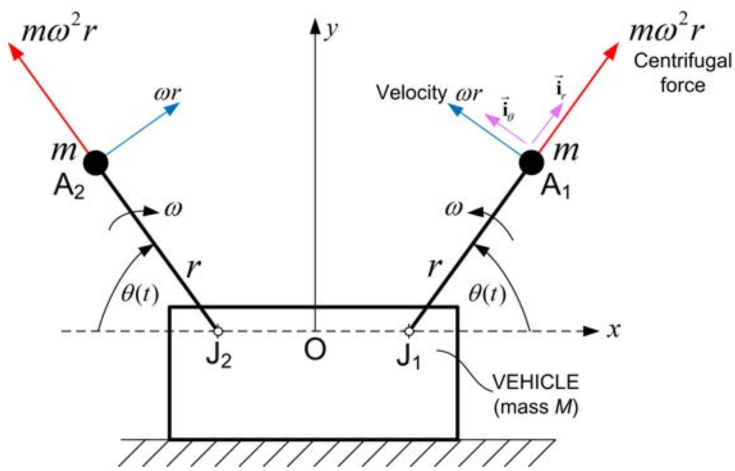

Figure 1. Contra-rotating masses.

By intuition, one may understand that when the angular velocity is adequately high, the *centrifugal* forces ($-m\vec{a}$) can overcome the total weight when the arms are oriented upwards, and then they tend to lift the object which is taken off (provided it is not glued on the ground). It is remarkable that the overcoming of the total dead weight happens although the centrifugal force is internal to the system, i.e. is fictitious. To rephrase it, *although no external force is exerted on the system it is possible to take a lift*. Nevertheless, as we will see below, this fact in no way dictates that a net thrust is developed.

Similar observations, as well as experience gained by terrestrial devices, have motivated a lot of inventors to file patents, and a lot of prototypes have been built during the last 100 years. One of the most known patents is that by the USA citizen Norman Dean (1959) in which the contra-rotating masses move along circular paths, and thus the name "*Dean drive*" is very popular to the enthusiasts of inertial propulsion.

To derive the equations of motion we may follow any known methodology such as i) the application of Newton's laws working with free body diagrams, ii) the consideration of the center of mass, iii) the conservation of linear momentum, or iv) the derivation and solution of Lagrange's equations known from analytical mechanics. The easiest way is the first methodology, in which the Newton's second law is applied separately to each participating rigid body and finally the internal forces are eliminated (due to Newton's third law). All these four methodologies eventually lead to the same expression for the equations of motion.

For practical engineering purposes, we sometimes directly consider the fictitious inertial (*d' Alembert*) forces as if they were static forces and the problem becomes one of static analysis. Therefore, with the vehicle being on the ground surface, when the angular velocity $\omega(t)$ is a constant, the inertial (centrifugal) forces $m\omega^2 r$ lie along the radii J_1A_1 and J_2A_2, respectively, and are directed outwards as shown by red

color in Figure 1. Moreover, if the angular velocity $\omega(t)$ is not a constant, the vectors of the two inertial forces are not directed along the corresponding radii (because of the tangential inertial force $m\dot{\omega}r$ in the clockwise direction) but again they are symmetric with respect to the vertical y-axis. Therefore, due to the symmetry involved in the intentionally imposed contra-rotation, the horizontal forces are cancelled, and thus the result of this mechanism is to produce an oscillating inertial force resultant which is directed toward the vertical y-axis only (i.e., *unidirectional* total inertial force).
The external forces $\vec{F}_{ext}$ which are applied to the mechanical system of Figure 1 are as follows:

i) The two weights $m\vec{g}$ caused by the rotating masses plus the weight $M\vec{g}$ of the vehicle.
ii) The external force $\vec{F}_{reac}$ which the ground exerts to the vehicle.

Therefore, the algebraic value of $\vec{F}_{ext}$ in the y-direction will be:

$$F_{ext} = -(M+2m)g + F_{reac} \, . \tag{4}$$

Now, let us consider the unit vector along the radius (Figure 1):

$$\vec{\mathbf{i}}_r = \cos\theta\,\vec{\mathbf{i}} + \sin\theta\,\vec{\mathbf{j}}\,, \tag{5a}$$

as well as the unit vector in the tangential direction:

$$\vec{\mathbf{i}}_\theta = -\sin\theta\,\vec{\mathbf{i}} + \cos\theta\,\vec{\mathbf{j}}\,, \tag{5b}$$

where $(\vec{\mathbf{i}}, \vec{\mathbf{j}})$ are the unit vectors on the Cartesian axes *x* and *y*, respectively.
The resultant external force at the mass m at point A_1 (i.e., the internal force $\vec{T}$ applied to this mass *m* by the arm plus the weight $m\vec{g}$) equals the sum of the centripetal force:

$$\vec{F}_{cp} = -\left(m\omega^2 r\right)\vec{\mathbf{i}}_r \text{ (from } A_1 \text{ towards the joint } J_1\text{)}, \tag{6a}$$

plus, the tangential force

$$\vec{F}_t = \left(m\dot{\omega}r\right)\vec{\mathbf{i}}_\theta \text{ (towards the current rotation).} \tag{6b}$$

Considering Newton's second law on the mass *m*, we have

$$\vec{T} + m\vec{g} = \vec{F}_{cp} + \vec{F}_t \ . \tag{6c}$$

Hence, taking the opposite of Eq. (6c), we receive:

$$-\vec{T} = m\vec{g} \underbrace{-\left(\vec{F}_{cp} + \vec{F}_t\right)}_{F_{\text{inertial,m}}} = m\vec{g} + \vec{F}_{\text{inertial,m}} \, . \tag{6d}$$

Equation (6d) shows that the internal force $-\vec{T}$, which is exerted on the vehicle transmitted from the single mass m, equals the weight of this mass plus the *inertial force* $\vec{F}_{\text{inertial.m}}$ *itself*, with:

$$\begin{aligned} \vec{F}_{\text{inertial,m}} &\triangleq -\left(\vec{F}_{cp} + \vec{F}_t\right) \\ &= \left(m\omega^2 r\right)\vec{\mathbf{i}}_r - \left(m\dot{\omega}r\right)\vec{\mathbf{i}}_\theta . \end{aligned} \tag{6e}$$

Moreover, considering the couple of the two contra-rotating masses, the horizontal components of the vectors $\vec{F}_{\text{inertial,m}}$ are cancelled, while taking into account the direction cosines by Eq. 5(a,b), those force components in the y-direction eventually lead to the *total inertial force*:

$$F_{inertial} = 2mr(\omega^2 \sin\theta - \dot{\omega}\cos\theta)\,, \tag{7}$$

Obviously, the vertical reaction force F_{reac} from the ground surface applied to the vehicle will cancel the vehicle's weight $M\vec{g}$ plus the two symmetric internal forces $-\vec{T}$ (each given by Eq. (6d)), of which the normal projection onto the Cartesian y-axis equals $m\vec{g}$ plus the inertial force $F_{inertial}$ [given by Eq. (7)]. Therefore, we have:

$$\begin{aligned}
F_{reac} &= Mg + 2\left(\vec{T}\cdot\vec{\mathbf{j}}\right) \\
&= Mg + 2\left(-m\vec{g} - \vec{F}_{\text{inertial,m}}\right)\cdot\vec{\mathbf{j}} \\
&= Mg + 2mg - 2\left(\vec{F}_{\text{inertial,m}}\cdot\vec{\mathbf{j}}\right) \\
&= \left(M+2m\right)g - 2\left[\left(m\omega^2 r\right)\vec{\mathbf{i}}_r - \left(m\dot{\omega} r\right)\vec{\mathbf{i}}_\theta\right]\cdot\vec{\mathbf{j}} \\
&= \left(M+2m\right)g - 2\left(m\omega^2 r\right)\sin\theta + 2\left(m\dot{\omega} r\right)\cos\theta \\
&= \left(M+2m\right)g - \underbrace{2mr(\omega^2\sin\theta - \dot{\omega}\cos\theta)}_{F_{inertial}}
\end{aligned} \tag{8}$$

Clearly, Eq. (8) shows that the entire weight $\left(M+2m\right)g$ is transferred to the reaction force, but the latter is released by the inertial force which is subtracted.

Eliminating the weights between Eq. (8) and Eq. (4), the general expression of the total external force on the system becomes:

$$F_{ext} = -2mr(\omega^2\sin\theta - \dot{\omega}\cos\theta)\,. \tag{9}$$

Comparing Eq. (7) with Eq. (9), one may validate that:

$$F_{ext} = -F_{inertial}\,. \tag{10}$$

Obviously, Eq. (10) is closely related to the condition:

$$F_{ext} + F_{inertial} = 0\,, \tag{11}$$

which is the analog of Eq. (2) for the totality of the three mass particles (m, m, M), i.e., comes directly from Newton's second law. In other words, when the vehicle lies on the ground surface, the totality of the previously mentioned external forces F_{ext} equals the opposite of the inertial forces ($-F_{inertial}$).

Obviously, setting $\sin\theta = 1$ (i.e. $\theta = 90^0$ degrees, which is associated to the arms J_1A_1 and J_2A_2 in the upward vertical direction), Eq. (8) suggests that when the magnitude of the angular velocity is small enough, the algebraic value of the reaction force F_{reac} is positive which means that the inertial force cannot overcome the total weight, and thus the vehicle remains in contact with the ground by compressing it. In contrast, when the angular velocity progressively increases becoming adequately large, it produces inertial force larger than the total weight associated to a zero reaction force (negative values are not allowed unless the vehicle is glued on the ground), and thus the whole mechanism may take a lift performing a vertical shot (upward jump) in the positive y-direction. Details about vehicle's motion are given at the end of section "Role of linear momentum."

For the sake of brevity, we assume that the vehicle stands on the horizontal ground surface and is hold immobilized by the assistance of a mechanical hand. Then, although we present a general theory, we particularly focus on two characteristic cases regarding the initial velocity $V_{CM,0}$ at the centre of mass, as follows:

i) The maximum initial velocity of the rotating masses occurs when $\theta_0 = 0$ (connecting arms horizontal, and thus $V_{CM,0} \neq 0$), i.e. at a position at which the vertical component of the centrifugal force vanishes.

ii) When $\theta = 90^0$ (connecting arms vertical, and thus $V_{CM,0} = 0$), the velocity component of the rotating masses in the vertical y-direction vanishes while the total centrifugal force becomes maximum.

As we shall see below, it is the initial velocity $V_{CM,0}$ at the centre of mass which controls the vehicle's motion.

Equivalence of Dean-drive with the mass-spring system

It is also worthy to notice that the above observations regarding the upward jump, are also valid for a vertically oriented spring-mass system which, initially, is highly compressed on the ground surface, and then is left free to oscillate. From elementary mechanics, we know that when the mass velocity vanishes the spring force becomes maximum, while when the mass velocity takes its maximum value the spring force vanishes (Halliday and Resnick 1966). In other words, the notorious Dean drive is very similar to a spring-mass system, in the sense that the perpendicular projection of the two rotating masses onto the vertical *y*-axis oscillate like the mass in the spring-mass system of one degree of freedom (i.e., with displacement amplitude double of the arms' radius: $2\delta_0 = 2r$), as shown in Figure 2. As soon as the vehicle stands on the ground there is no difference between the reaction forces exerted on the ground in the two systems, i.e., the Dean drive (Figure 2a) and the mass-spring system (Figure 2b) provided $\omega = \sqrt{k/m}$. Clearly, when the spring is decompressed and further elongated by half-amplitude δ beyond the equilibrium length l_0, the spring undertakes a tensile force and thus it pulls the ground up. Therefore, if the end of the spring is not glued on the ground the spring will jump upwards.

The only difference between Dean drive and the spring-mass system, occurs when the vehicle takes off the ground. Then, in the Dean-drive the displacement amplitude may be preserved *constant* (equal to $2r$) due to the continuous energy supply by the motors at points J_1 and J_2, which in combination with the rigid connecting arms J_1A_1 and J_2A_2 can keep the angular momentum ω be constant ($\omega = \omega_0$). In contrast, when the mass-spring system takes off under the same conditions, the motion is fully controlled by the energy conservation. In both cases the maximum vertical travel depends on the initial velocity $V_{CM,0}$ at the centre of mass of the corresponding system.

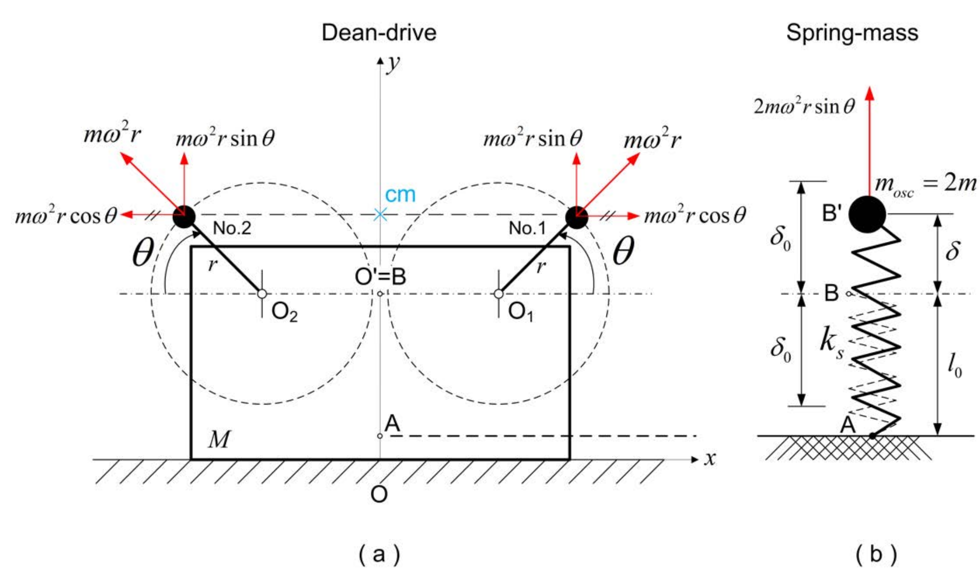

Figure 2. Equivalence between (a) Dean drive and (b) spring-mass system.

Questions posed

Having said this, aiming at implementing these concepts as a propulsion means, there are at least two questions that have kept inventors busy (we start with the most important question) which will be answered later:

- After the vehicle reaches the maximum height in the first jump (because of a few full rotations which form the *first cycle* of motion), is it possible to repeat the same procedure in a second cycle of motion to obtain a still higher height?
- Is it possible to maximize the effect of the inertial forces per cycle?

Inertially propulsed vehicle in the gravitational field

For the sake of generality, in this subsection we assume a variable angular velocity $\omega(t)$, which is controlled by two fully synchronized motors at the points J_1 and J_2. When inertial forces have lifted the vehicle off the ground (as was explained in the section "Inertial drive and reaction force"), the equations of motion change somewhat. In more detail, an observer on the vehicle who is positioned between the points J_1 and J_2 will see the masses rotating on circles (relative motion), but also participates in the acceleration $\ddot{y}_M$ of the vehicle. Therefore, not only the above relative inertial force will continue to exist but also the value $-m\ddot{y}_M\vec{\mathbf{j}}$ must be added to it.

To avoid any ambiguity about the above claim, the y-coordinate of the rotating mass with respect to the ground surface, here denoted by y_m, is written in terms of vehicle's attitude y_M and polar angle θ, as follows:

$$y_m = y_M + r\sin\theta \tag{12}$$

Taking the second derivative of y_m with respect to time t, we obtain:

$$\ddot{y}_m = \ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) \tag{13}$$

Eq. (13) shows that the y-component of the mass absolute acceleration equals the sum of the y-component of centripetal and tangential accelerations (relative terms) plus the acceleration of the moving observer on the vehicle. Therefore, considering that $\omega = \dot{\theta}$ and $\dot{\omega} = \ddot{\theta}$, the y-component of a single mass is given as

$$\begin{aligned} F^y_{\text{inertial,m}} &= -m\ddot{y}_m \\ &= -\ddot{y}_M + mr\left(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right) \\ &= -\ddot{y}_M + mr\left(\omega^2\sin\theta - \dot{\omega}\cos\theta\right), \end{aligned} \tag{14}$$

which is the generalization of Eq. (6e).

If we work in a similar way as in section "Inertial drive and reaction force", but now considering only the y-component, then Newton's second law applied on the single mass m dictates that the internal force T_y (from the arm to a single mass m) fulfils the condition:

$$T_y - mg = m\left[\ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right], \tag{15a}$$

whence for the two masses will be:

$$-2T_y = -2mg - 2m\left[\ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right]. \tag{15b}$$

Since the vehicle of mass m undertakes $-2T_y$ from both rotating masses through the rotating arms, using Eq. (15b) it comes out that the total external force exerted on the vehicle will be:

$$-Mg - 2T_y = -Mg - 2mg - 2m\left[\ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right]$$
$$= -(M+2m)g - 2m\left[\ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right] \quad . \qquad (16a)$$

The application of Newton's second law on the vehicle using the external force given by Eq. (16a) gives:

$$-(M+2m)g - 2m\left[\ddot{y}_M + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\right] = M\,\ddot{y}_M \quad . \qquad (16b)$$

Arranging the terms in Eq. (16b) in combination with Eq. (7), we eventually receive the following differential equation of vehicle's motion in vacuum due to the two contra-rotating masses:

$$\ddot{y}_M = \frac{2mr}{(M+2m)}\left(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right) - g$$
$$= \frac{F_{\text{inertial}}}{(M+2m)} - g \quad . \qquad (16c)$$

For the sake of shortness, we introduce the constant

$$\lambda = \frac{2mr}{(M+2m)} \ , \qquad (16d)$$

which facilitates the handling of the *two-body system* (the contra-rotating masses are equivalent to one mass of magnitude 2*m*, while the second mass is *M* of the vehicle).

The first time-integration of Eq. (16c) gives an analytical expression for vehicle's velocity $\dot{y}_M$:

$$\dot{y}_M(t) = \int_0^t \frac{2mr}{(M+2m)}\left(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right)\mathrm{d}\tau - gt + v_{M0}$$
$$= \frac{1}{(M+2m)}\cdot\underbrace{2mr\int_0^t\left(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right)\mathrm{d}\tau}_{I(t)} - gt + v_{M0}$$
$$\equiv \lambda\cdot\int_0^t\left(\omega^2\sin\theta - \dot{\omega}\cos\theta\right)\mathrm{d}\tau - gt + v_{M0}$$
$$= \lambda\cdot\left[-\omega\cos\theta\right]_{\theta_0}^{\theta} - gt + v_{M0}$$
$$= -\lambda\cdot\left(\omega\cos\theta - \omega_0\cos\theta_0\right) - gt + v_{M0}. \qquad (16e)$$

Note that the quantity

$$I(t) \equiv \int_0^t F_{inertial}\,\mathrm{d}\tau$$
$$= \int_0^t 2mr\left(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta\right)\mathrm{d}\tau, \qquad (16f)$$

which is involved in the second equality of Eq. (16e), is called ***impulse*** of the inertial forces (the denominator was excluded to be consistent with the previous Eq. (7). As was shown in Eq. (16e), the impulse function is eventually given by the formula:

$$I(t) = -2mr\left(\omega\cos\theta - \omega_0\cos\theta_0\right), \qquad (16g)$$

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while the vehicle's velocity is given by

$$\dot{y}_M(t) = \frac{I(t)}{(M+2m)} - gt + v_{M0} \tag{16h}$$

Remark on the impulse of inertial force

If we isolate the two rotating masses, the sum of the external forces on them (i.e. $2T_y$ from the arms and the total weight $-2mg$) equals to the centripetal force, which by definition is opposite to the centrifugal force. Therefore, the change of the linear momentum for the system of $2m$ equals to the impulse of the centrifugal force, i.e. is opposite to the impulse of the inertial (centrifugal) force:

$$\Delta P(t) = -\int F_{inertial} \mathrm{d}\tau \equiv -I(t) \tag{16i}$$

where

$$\Delta P(t) = m\omega r\left(\cos\theta(t) - \cos\theta_0\right). \tag{16j}$$

So far, a lot of inventors have dealt with the abovementioned quantity *I(t)* within a rotation (period), trying to produce a positive magnitude which will be opposed to the negative figure of the free fall velocity $-gt$, shown in the right part of Eq. (16h). This happens because behind their mind is the erroneous assumption that the initial force $F_{inertial}$ operates as an external force. In some of these patents, it is probable that a non-zero impulse value may be received, and sometimes it has been claimed to have been measured. In this paper, we shall clearly show that such a non-zero (positive or negative) quantity of course may be produced for sure but, unfortunately, this factoid is incapable of overcoming the gravity term $-gt$.

The reason for zero impulse in two characteristic cases (constant and periodic angular velocity) is as follows:

- In case of *constant* angular velocity, i.e. $\omega = \omega_0 = \text{const.}$, every 360 degrees we have $\left(\cos\theta = \cos\theta_0\right)$ and thus the quantities $\left(\omega\cos\theta - \omega_0\cos\theta_0\right)$ in Eq. (16g) as well as ΔP in Eqs. (16i,j) vanish, that is $I(T) = I(2T) = \cdots = I(nT) = 0$, where T is the period. As a result, no impulse is produced at the end of any period. Moreover, for any intermediate time $0 < t < T$ we have $I(t+T) = I(t)$.
- In *periodic* repeated motion, even if $\omega(t)$ varies in time, every 360 degrees we have $\omega = \omega_0$ (due to periodicity) and again $\left(\cos\theta = \cos\theta_0\right)$. Therefore, the quantity $\left(\omega\cos\theta - \omega_0\cos\theta_0\right)$ in Eq. (15f) vanishes, and thus no net impulse is produced again (i.e., $\Delta P = -I = 0$).

In the most general case of an *arbitrary* variable angular velocity $\omega(t)$ we work as follows. First we calculate the impulse between the initial polar angle θ_0 and the vertical upper position of the arms at $\theta = \pi/2$ (with $\cos\theta = 0$), which according to Eq. (16g) is $I_0 = 2mr\omega_0\cos\theta_0$. Then, we split the rotation in equal angular intervals each of length π (180 degrees), starting at the aforementioned upper point and terminating to a lower one, as follows: $\theta \in \left[\pi/2, 3\pi/2\right]$, $\theta \in \left[3\pi/2, 5\pi/2\right]$, $\theta \in \left[5\pi/2, 7\pi/2\right]$ etc. Since at the extreme points of these intervals we have $\cos\theta_{\text{extreme}} = 0$, according to Eq. (16g) the differential impulse (time integral) from the upper point to the lower one *vanishes*. Since many rotations form a set of intervals $\theta \in \left[\pi/2, 3\pi/2\right]$, $\theta \in \left[3\pi/2, 5\pi/2\right]$, $\theta \in \left[5\pi/2, 7\pi/2\right]$, and so on, it is obvious that the total impulse from $\theta = \theta_0$ till the upper position ($\theta = 2k\pi + \pi/2$) restricts to the magnitude $I_0 = 2mr\omega_0\cos\theta_0$.

Continuing the above thoughts regarding the arbitrary angular velocity $\omega(t)$, it is more accurate to calculate the impulse exactly at the end of a full rotation, i.e., from θ_0 to $\theta_0 + 2\pi$. Applying again Eq. (16g), we receive:

$$\begin{aligned} I(T) &= -2mr\left(\omega\cos(\theta_0+2\pi)-\omega_0\cos\theta_0\right) \\ &= -2mr\left(\omega(T)-\omega_0\right)\cos\theta_0 \\ &= -2mr\Delta\omega\cos\theta_0, \end{aligned} \tag{16k}$$

where $\Delta\omega$ is the change of angular velocity within a full rotation of the arms. Equation (16k) justifies the possibility to obtain nonzero impulse of inertial forces within a full rotation of the arms, simply by changing the angular velocity at the end of each rotation ($\Delta\omega \neq 0$).

Further integration of equations of motion

By further integration of Eq. (16e) in time, now written as

$$\dot{y}_M = \frac{1}{(M+2m)}\cdot I(t) - gt + v_{M0}\ , \tag{16l}$$

we obtain the closed-form expression of the altitude function $y_M(t)$ given by

$$y_M(t) = \left(y_{M0} + v_{M0}t - \frac{1}{2}gt^2\right) + \underbrace{\left(\lambda\omega_0\cos\theta_0\right)t - \lambda\left(\sin\theta - \sin\theta_0\right)}_{\int_0^t I(\tau)\mathrm{d}\tau}, \tag{17}$$

where y_{M0} is the initial position (height) of the vehicle, v_{M0} its initial velocity, ω_0 the angular velocity at $t = 0$, and θ_0 the initial polar angle at $t = 0$. It is clearly shown in Eq. (17) that, apart from the gravitational terms, vehicle's height is also influenced by the integral of the impulse of inertial forces. In the lack of a concrete name of this quantity in classical mechanics, in this paper we baptize $\int_0^t I(\tau)\mathrm{d}\tau$ as "*cumulative impulse*".

In general, one may observe three terms in the right-hand side of Eq. (17) as follows:

1) The first term, $\left(y_{M0} + v_{M0}t - 1/2\,gt^2\right)$, is the classical expression for the vertical upward shot (or the free fall) of a particle in Earth's gravitational field of constant acceleration g.
2) The second term [$\left(\lambda\omega_0\cos\theta_0\right)t$, linear in time] is of major importance, because the factor $\left(\lambda\omega_0\cos\theta_0\right)$ is the only one that cancels the free fall, of course at a certain extent. It is highly dependent on the cosine of the initial polar angle θ_0 at time $t = 0$, i.e. when the vehicle takes off the ground surface. Unfortunately, whatever the magnitude of the factor $\left(\lambda\omega_0\cos\theta_0\right)$ is, in the progress of time t the quadratic term $-1/2\,gt^2$ will eventually dominate.
3) The third term is bounded, i.e. $\left|\lambda\left(\sin\theta - \sin\theta_0\right)\right| \le 2\lambda$, and thus its role in the maximum allowable height $y_{M.\max}$ is rather minor.

Although the above procedure is strictly mathematical and does not leave any space for ambiguities, it may be desirable to study the physics dealing with the conservation of the linear momentum in the mechanical system as well.

Maximum height and required time

The determination of the maximum vertical vehicle's travel is a nonlinear problem, but a first-order estimation may be performed by ignoring the small term $\lambda\left(\sin\theta - \sin\theta_0\right)$ in Eq. (17). Equating the first derivative of the function $y_M(t)$ to zero, we receive that the raising time is given as:

$$t_{rise} \cong \frac{\lambda\omega_0\cos\theta_0}{g} \tag{18}$$

By substituting Eq. (18) into Eq. (17) and ignoring $\lambda\left(\sin\theta - \sin\theta_0\right)$, for the given initial angle θ_0 the ultimate vehicle's height is approximated as:

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$$y_{M,\max} \cong \frac{(\lambda\omega_0 \cos\theta_0)^2}{2g}. \tag{19}$$

In other words, the system 'vehicle + rotating masses' is governed by the initial velocity V_{cm} of its *center of mass*, which at time $t = 0$ is given as

$$V_{cm} = \frac{M \cdot 0 + 2m \cdot (\omega_0 r \cos\theta_0)}{(M + 2m)} = \frac{2mr}{(M + 2m)}(\omega_0 \cos\theta_0) = \lambda\omega_0 \cos\theta_0. \tag{20}$$

Substituting Eq. (20) into Eq. (19), one may recognize the well-known formula of classical physics applied to a single mass particle in vertical shot, $y_{\max} = V_0^2/(2g)$, which in our case may be applied to the center of mass (Halliday and Resnick, 1966).

Important remark: Equation (19) shows that the global maximum of vehicle's height is produced when $\cos\theta_0 = 1$, which means that the arms are oriented in the horizontal direction ($\theta_0 = 0$) and rotate upwards.

The role of linear momentum

Now, let us follow a different way to extract the equations of motion, which is based on the utilization of total linear momentum of the system.

Obviously, when the vehicle is immobile on the ground surface ($V_{M0} = 0$) and the polar angle of the connecting arm is θ_0, the initial total linear momentum in the *y*-direction is given by

$$P_0 = M \cdot 0 + 2m \cdot \omega r \cdot \cos\theta_0. \tag{21}$$

Obviously, if the vehicle is fixed on the ground and the angular velocity increases, then the linear momentum P_0 increases as well. To avoid any ambiguity, here we assume that when the conditions (ω, θ_0, P_0) appear at time *t = 0*, the ground surface is suddenly withdrawn (opens like the cover of a well) and the system is found in the vacuum. Therefore, at the later arbitrary time instance *t > 0* the total linear momentum is:

$$P_t = M \cdot V_M + 2m \cdot (V_M + \omega r \cdot \cos\theta) \tag{22}$$

On the other hand, according to Newton's second law in terms of linear momentum, the change of system's linear momentum equals the impulse of the gravitational forces (the only external forces), that is:

$$\Delta P = P_t - P_0 = \int_0^t [-(2m + M)g]\,\mathrm{d}\tau = -(2m + M)gt \tag{23}$$

Combining Eqs. (21) to (23), we receive:

$$V_M(t) = -gt - \lambda\omega \cdot (\cos\theta - \cos\theta_0) \tag{24}$$

Considering the definition of velocity

$$V_M = \frac{dy_M}{dt}, \tag{25a}$$

also written as

$$\begin{aligned} V_M &= \underbrace{\frac{2m\omega r}{M + 2m}}_{\lambda\omega}(\cos\theta_0 - \cos\theta) - gt \\ &= \lambda\omega(\cos\theta_0 - \cos\theta) - gt, \end{aligned} \tag{25b}$$

after integration we receive:

$$y_M(t) = -\frac{1}{2}gt^2 + \left[\lambda\omega\cos\theta_0\right]t - \lambda\left(\sin\theta - \sin\theta_0\right) \quad (26)$$

One may observe that, starting from impulse (linear momentum) considerations, we eventually received the same expressions which had been previously derived using Newton's second law, i.e. Eq. (17) with $y_{M0} = 0, v_{M0} = 0$.

Although the system in vacuum is open, it is instructive to see what happens if we temporarily neglect the influence of the gravitational field g (short-time *semi-closed* system). Assuming a constant angular velocity $\omega = \theta/t$, and that at the initial time $t = 0$ the arms are horizontally positioned (i.e., $\theta_0 = 0$), the time instants for $\theta = 90^0, 180^0, 270^0, 360^0$ will be $t = \pi/2\omega, \pi/\omega, 3\pi/2\omega, 2\pi/\omega$, respectively. Applying Eq. (24) and Eq. (26) in conjunction with $g = 0$, we obtain the results shown in Table 1.

The essential information shown in Table 1 is that:

1) At the positions $\theta = 0^0, 360^0$ in which the arms are horizontal and rotate *upwards*, the vehicle's velocity vanishes. In other words, the total linear momentum is fully undertaken by the two rotating masses m.
2) At the positions $\theta = 90^0, 270^0, 450^0, 630^0$ in which the arms are vertical and thus the vertical component of mass m velocity vanishes, the whole linear momentum is fully undertaken by the vehicle, and hence its vertical velocity becomes $\lambda\omega$.
3) At the positions $\theta = 180^0, 540^0$ in which the arms are horizontal and rotate *downwards*, the vehicle's velocity duplicates, i.e., its velocity becomes $2\lambda\omega$. In this way, the conservation of linear momentum is ensured.
4) Regarding the vertical travelled distance y_M, it increases monotonically with time t, by the amount of $\lambda\pi$ for every 180 degrees.

Table 1. Velocity and position of the vehicle for the first two rotations assuming initial polar angle $\theta_0 = 0$, in the hypothetical case of momentum conservation (neglected gravity).

Rotation #1				**Rotation #2**			
t [s]	θ [deg]	V_M [m/s]	y_M [m]	t [s]	θ [deg]	V_M [m/s]	y_M [m]
0	0	0	0	$\frac{2\pi}{\omega} + 0$	360	0	$2\lambda\pi + 0$
$\frac{\pi}{2\omega}$	90	$\lambda\omega$	$\lambda\left(\frac{\pi}{2} - 1\right)$	$\frac{2\pi}{\omega} + \frac{\pi}{2\omega}$	450	$\lambda\omega$	$2\lambda\pi + \lambda\left(\frac{\pi}{2} - 1\right)$
$\frac{\pi}{\omega}$	180	$2\lambda\omega$	$\lambda\pi$	$\frac{2\pi}{\omega} + \frac{\pi}{\omega}$	540	$2\lambda\omega$	$2\lambda\pi + \lambda\pi = 3\lambda\pi$
$\frac{3\pi}{2\omega}$	270	$\lambda\omega$	$\lambda\left(\frac{3\pi}{2} + 1\right)$	$\frac{2\pi}{\omega} + \frac{3\pi}{2\omega}$	630	$\lambda\omega$	$2\lambda\pi + \lambda\left(\frac{3\pi}{2} + 1\right)$
$\frac{2\pi}{\omega}$	360	0	$2\lambda\pi$	$\frac{2\pi}{\omega} + \frac{2\pi}{\omega}$	720	0	$2\lambda\pi + 2\lambda\pi = 4\lambda\pi$

Remark: Let us consider the extreme case in which $\theta_0 = \pi/2$ at time $t_0 = 0$, which means that the connecting arms are upwards and vertical, a fact that means that on the y-axis the normal projection of the mass velocity vanishes and the centrifugal force becomes maximum. If the angular velocity fulfils the condition $2m\omega^2 r > (2m + M)g$, the support force vanishes, and the system takes off. From a different point of view, a short time later ($t > t_0 = 0$) the polar angle becomes $\pi > \theta > \theta_0 = \pi/2$ and thus the projection of the mass velocities on the y-axis take a negative value. Following the above concept of momentum conservation (for negligible g), the negative velocity of the rotating masses $2m$ is cancelled by the upward positive vehicle's velocity (micro-motion). In other words, the fact of the large centrifugal force which can cancel the weight is fully consistent with the principle of the conservation of the linear momentum in the y-direction. Nevertheless, since the initial velocity of the center of mass is zero, it eventually falls.

Center of mass

The motion of the center of mass (CM) is of major interest in this study. As usual, the vertical position of the CM is given as

$$y_{CM} = \frac{My_M + 2my_m}{M + 2m}. \tag{27}$$

Obviously, when the vehicle is detached from the ground surface ($F_{reac} = 0$), the only external force exerted on the system is the total weight $-(M + 2m)g$. Then, Newton's second law suggests that

$$(M + 2m)\ddot{y}_{CM} = -(M + 2m)g, \qquad \text{whence } \ddot{y}_{CM} = -g \tag{28}$$

Below we shall show that the inertial force $F_{inertial}(t)$ determines a relationship between the motion of the CM, $y_{CM}(t)$, and the position of the vehicle, $y_M(t)$. Indeed, taking the second time derivative in Eq. (27) and also considering the previous relationship $y_m = y_M + r\sin\theta$, the desired relationship is:

$$\begin{aligned}(M + 2m)\ddot{y}_{CM} &= (M + 2m)\ddot{y}_M - F_{inertial}(t) \\ &= -(M + 2m)g.\end{aligned} \tag{29}$$

Equation (29) clearly shows that the inertial force $F_{inertial}(t)$ is merely the difference between these two quantities, i.e.,

$$F_{inertial}(t) = (M + 2m)\left(\ddot{y}_M - \ddot{y}_{CM}\right) \tag{30}$$

Modifications of the Dean drive

For the conventional Dean drive, the pattern of the inertial force $F_{\text{inertial}}(t)$ toward the y-direction is illustrated in Figure 3 with the solid blue line. This case concerns initial angle $\theta_0 = 0$ and period $T = 0.02\,\text{s}$. The part AB refers to upward motion, while BC downward motion. In the original Dean drive in which the two contra-rotating are connected to the vehicle through rigid arms of constant radius r, the sinusoidal curve BC is *symmetric* to AB with respect to the point B.
Therefore, the integral becomes $\int_0^T F_{\text{inertial}}(t)\,\mathrm{d}t = 0$, i.e. it vanishes.

As previously mentioned, a lot of people expect to *break the symmetry* by shortening the magnitude of the negative part BC such as that shown by the red dashed line in Figure 3. If such a supposed "polarization" of the inertial force was possible, a net upward force (net thrust) would appear, and thus a continuous propulsion would be possible. This is the central unsolved issue of the inertial propulsion.

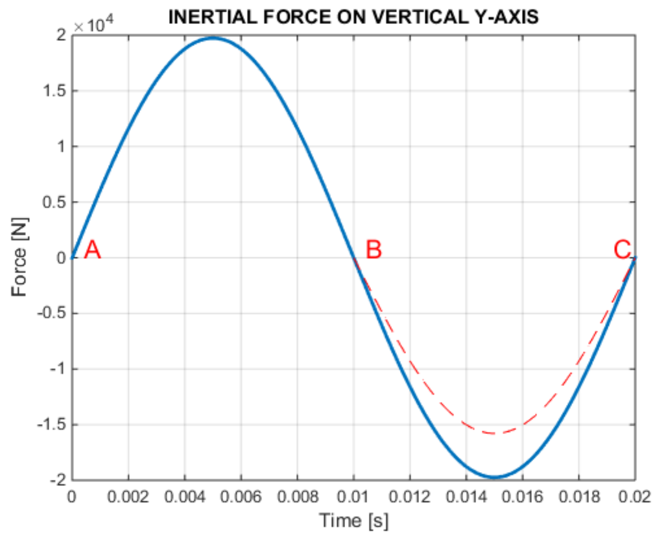

Figure 3. Variation of inertial force *versus* time and desired polarization/rectification.

In this section we will study two cases regarding the possible modifications of Dean drive. The first case (Non-circular path) refers to the modification in the shape of the traced curve, from an ideal circle to a different one. The second case (Shifted joints) concerns joint points which can slide along the vehicle.

Non-circular path

The general case

In the most general case, for each moving mass Newton's second law gives:

$$T_y - mg = m\ddot{y}_m \ , \tag{31a}$$

whence for the two mases the total force transmitted to the vehicle will be:

$$-2T_y = -2mg - 2m\ddot{y}_m \ . \tag{31b}$$

Furthermore, the application of Newton's second law on the vehicle gives:

$$-Mg - 2T_y = M\ddot{y}_M \ .$$

or by virtue of Eq. (31b):

$$-(M+2m)g - 2m\ddot{y}_m = M\,\ddot{y}_M \, . \tag{31c}$$

By once integrating Eq. (31c) in terms of t, we receive:

$$\dot{y}_M(t) \equiv v_M(t) = -\frac{(M+2m)}{M}gt - 2\frac{m}{M}\left(\dot{y}_m(t) - \dot{y}_{m0}\right) + v_{M0} \ . \tag{31d}$$

A further integration of Eq. (31d), where $v_M \triangleq \mathrm{d}y_M/\mathrm{d}t$, leads to the position of the vehicle at:

$$y_M(t) = -\left[\frac{(M+2m)g}{2M}\right]\cdot t^2 + \left(\frac{2m}{M}\dot{y}_{m0} + v_{M0}\right)\cdot t - \frac{2m}{M}y_m(t) + \left(\frac{2m}{M}y_{m0} + y_{M0}\right) \quad (31e)$$

Equation (31e) shows that the linear term is exceeded by the quadratic term in the square brackets, and thus the vehicle cannot monotonically move upwards. The only unclear point in this argumentation is whether it is possible to determine a possible function $y_m(t)$ which is the third term in the right-hand side of Eq. (31e), through a mechanical mechanism (such as piston-crank, three-bar, etc.), so as $y_M(t)$ always remains of positive magnitude. To make this hypothesis possible it should be:

$$y_m(t) < -\left[\frac{(M+2m)g}{4m}\right]\cdot t^2 + \left(\dot{y}_{m0} + \frac{M}{2m}v_{M0}\right)\cdot t + \left(y_{m0} + \frac{M}{2m}y_{M0}\right) \quad (32)$$

Unfortunately, such a function $y_m(t)$ will tend to $-\infty$ when $t\to\infty$, which means that it is not possible that the masses of magnitude m are connected to the vehicle using a proper kinematic mechanism. The real meaning of Eq. (32) is that the rotating masses must be ejected from the vehicle, which in such a case will operate as a usual action-reaction system.

Fixed centre of rotation with variable telescopic radius

Among several possibilities of a non-circular path, we study the case of joints (J1, J2) fixed to the vehicle (as in Figure 1), but with variable radius (telescopic system), so as

$$y_m(t) = y_M(t) + r(t)\sin\theta(t) \quad (33)$$

In such as case the generalized equation of vehicle's motion becomes:

$$(2m+M)\ddot{y}_M + 2m(r\sin\theta)^{\cdot\cdot} + (2m+M)g = 0 \quad (34a)$$

or

$$\ddot{y}_M = -\frac{2m}{(2m+M)}(r\sin\theta)^{\cdot\cdot} - g \quad (34b)$$

The first integration of Eq. (34b) gives:

$$\dot{y}_M = (v_0 - gt) - \frac{2m}{(2m+M)}\left[\dot{r}(t)\sin\theta(t) + r(t)\dot{\theta}(t)\cos\theta(t) - (\dot{r}_0\sin\theta_0 + r_0\omega_0\cos\theta_0)\right] \quad (35)$$

The second integration implies:

$$\begin{aligned} y_M(t) = {} & \left(y_0 + v_0 t - \frac{1}{2}gt^2\right) \\ & - \frac{2m}{(2m+M)}\left\{\left[r(t)\sin\theta(t) - r_0\sin\theta_0\right] - (\dot{r}_0\sin\theta_0 + r_0\omega_0\cos\theta_0)t\right\} \end{aligned} \quad (36a)$$

or

$$y_M(t) = y_0 + \left[v_0 + \frac{2m}{(2m+M)}(\dot{r}_0 \sin\theta_0 + r_0\omega_0 \cos\theta_0) \right] t - \frac{1}{2}gt^2 - \frac{2m}{(2m+M)}\left\{\left[r(t)\sin\theta(t) - r_0 \sin\theta_0\right]\right\} \tag{36b}$$

One may observe in Eq. (36b) that the height $y_M(t)$ consists of:

- the usual term $\left(y_0 + v_0 t - 1/2\, gt^2\right)$ known from physics textbooks,
- the *linear* term $2m/(2m+M)\left(\dot{r}_0 \sin\theta_0 + r_0\omega_0 \cos\theta_0\right)t$, which is proportional to time t. Obviously, after a critical time ($t > t_{critical}$) it cannot withstand to the abovementioned *quadratic* term $-1/2\, gt^2$.
- the term $-2m/(2m+M)\left[r(t)\sin\theta(t) - r_0 \sin\theta_0\right]$, which needs to be commented.
  - If the radius $r(t)$ is bounded ($r < r_{\max}$), this term is also bounded and thus cannot cancel the continuously decreasing term $-1/2\, gt^2$.
  - If the radius $r(t)$ increases to infinity at a high velocity like a rocket, i.e. the two masses are *ejected* from the vehicle (say at $\sin\theta(t) = \sin\theta_0 = -1$), then this term is a positive figure which tends to infinity, and thus it is possible to cause propulsion to the vehicle. However, this case is only conventional physics based on the principle of action-reaction, and thus the concept cannot be used to build an inertial device.

Shifted joints

Let us also examine the issue pointed by Hampton (2022), which is also found cited in one of Dean's patents. We assume that the joints J_1 and J_2 in Figure 1 are not firmly attached to the vehicle as previously had been considered, but they can be controlled to harmonically slide on it according to the law

$$y_s = y_0 \cos(\omega_s t) \tag{37}$$

Therefore, the total absolute vertical distance of each rotating mass m will be

$$y_m = y_M + y_s + r\sin\theta \tag{38}$$

Taking the second derivative of $y_m(t)$ with respect to time, we have

$$\ddot{y}_m = \ddot{y}_M - \left(\omega_s^2 y_0\right)\cos(\omega_s t) + r\left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right) \tag{39}$$

The application of second Newton's law on the vehicle of mass M gives:

$$-(2m+M)g - 2m\ddot{y}_m = M\,\ddot{y}_M \tag{40}$$

Substituting Eq. (39) into Eq. (40), after rearrangement of the terms, one receives the following differential equation of motion:

$$\ddot{y}_M = -g - \frac{2m}{(2m+M)}\left[-\left(\omega_s^2 y_0\right)\cos(\omega_s t)\right] - \frac{2mr}{(2m+M)}\left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right) \tag{41}$$

Obviously, Eq. (41) is an extension of Eq. (16c), and thus a similar approach is followed. The first integration of Eq. (41) gives an analytical expression for vehicle's velocity $\dot{y}_M$:

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$$\dot{y}_M(t) = v_{M0} - gt + \frac{2m\omega_s^2 y_0}{(M+2m)} \int_0^t \cos(\omega_s \tau)\,\mathrm{d}\tau + \frac{2mr}{(M+2m)} \cdot \int_0^t \left(\dot{\theta}^2 \sin\theta - \ddot{\theta}\cos\theta\right)\mathrm{d}\tau$$
$$\equiv v_{M0} - gt + \frac{2m\omega_s^2 y_0}{(M+2m)} \left[\frac{\sin(\omega_s \tau)}{\omega_s}\right]_0^t + \lambda \cdot \int_0^t \left(\omega^2 \sin\theta - \dot{\omega}\cos\theta\right)\mathrm{d}\tau$$
$$= v_{M0} - gt + \frac{2m\omega_s y_0}{(M+2m)} \sin(\omega_s t) - \lambda \cdot \left(\omega\cos\theta - \omega_0 \cos\theta_0\right) \tag{42}$$

A second integration provides the distance y_M between the ground surface and the vehicle:

$$y_M(t) = \underbrace{\left(y_{M0} + v_{M0}t - \frac{1}{2}gt^2\right)}_{\text{standard shot}} - \underbrace{\frac{2my_0}{(2m+M)}(\cos\omega_s t - 1)}_{\text{shifted joints}} + \underbrace{(\lambda\omega_0\cos\theta_0)t - \lambda(\sin\theta - \sin\theta_0)}_{\text{inertial force}} \tag{43}$$

Equation (43) shows that the vehicle performs an average motion which is that of a standard shot, accompanied by three additional terms as follows. The first is a bounded small term due to the shifted joints. The second is a linear term $(\lambda\omega_0 \cos\theta_0)t$ coming from the initial velocity of the system, and the third is another bounded term of absolute value less than 2λ; note that these last two terms were previously found (when the joints were firmly fixed on the vehicle).

Therefore, the shifted joints lead to a harmonic term of amplitude $2m/(2m+M)\,y_0$, which is only a *portion* of the imposed amplitude y_0 at the joints J_1 and J_2 on the vehicle. In this context, within the gravitation field, the effect of the shifted joints to the maximum altitude of the vehicle is minor.

Numerical results

Let us consider the following data:

Rotating mass : $m = 1\,\text{kg}$

Vehicle mass : $M = 5\,\text{kg}$

Radius : $r = 0.1\,\text{m}$

Angular velocity: $\omega = 314.16\,\text{s}^{-1}\,(3000\,\text{rpm})$

Initial polar angle : $\theta_0 = 0\,\text{deg}$

Gravitational acceleration: $g = 9.81\,\text{m/s}^2$

Three cases with the same initial angular velocity $\omega_0 = 314.16\,\text{s}^{-1}\,(3000\,\text{rpm})$ and the same initial angle $\theta_0 = 0\,\text{deg}$ will be studied below, as summarized in Table 2. By selecting a few $n_{\text{rotations}}$ rotations (each of 360 degrees), we subdivide each rotation into (say) 360 equal angular segments and then use Table 2 to determine the corresponding time instants t.

Table 2. Models to be studied.

Angular velocity	$\theta(t)$	$\omega(t)$	t
Constant	$\theta_0 + \omega_0 t$	ω_0	$t = (\theta - \theta_0)/\omega_0$
Exponential	$\theta_0 + \frac{e^{at} - 1}{a}\omega_0$	$\omega_0 e^{at} = \omega_0 + a(\theta - \theta_0)$	$t = \frac{1}{a}\ln\left[\frac{a(\theta-\theta_0)}{\omega_0} + 1\right]$
Linear	$\omega_0 + at$	$\theta_0 + \omega_0 t + \frac{at^2}{2}$	Root of binomial $at^2 + (2\omega_0)t + 2(\theta_0 - \theta) = 0$

Constant angular velocity

We start considering a constant angular velocity $\omega = \omega_0 = 314.16\,s^{-1}\,(3000\,\text{rpm})$. In Figure 4 we present the application of Eq. (17) for two full rotations, with and without the influence of the gravitation. In the first case (with gravity) the linear momentum is not preserved and thus the height is slightly smaller (blue line), while in the second (no gravity) it is preserved, and the height is slightly larger (red line). One may also observe that at the end of the second rotation the deviation between the two cases is larger than what it is at the end of the first rotation.

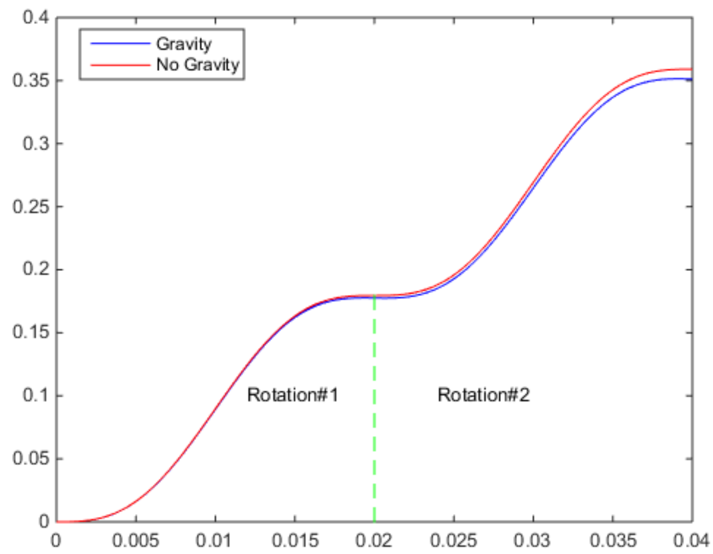

Figure 4. Time history of vehicle's lifting height.

To show the influence of the gravitation in long term, in Figure 5 we extend the presentation of vehicle's height versus time for the first 100 rotations. One may observe the significant discrepancy between gravity and non-gravity conditions. In the former case the vehicle's velocity is almost constant (slightly oscillating around a constant value) while in the latter case it oscillates around the well-known parabolic curve drawn by the center of mass.

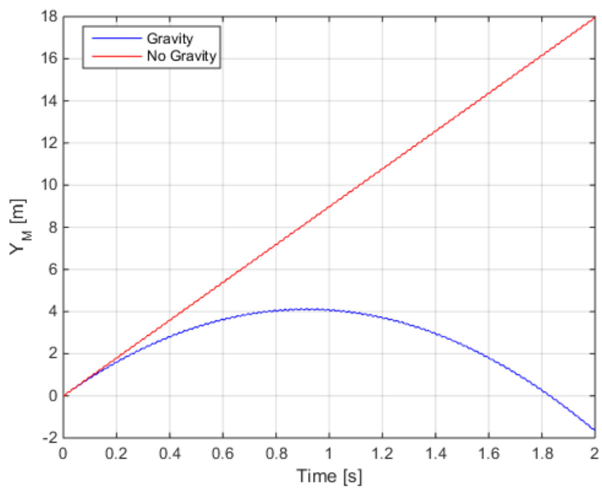

Figure 5. Time history of vehicle's lifting height for the first 100 rotations.

In addition, it is interesting to study the time history of vehicle's velocity according to Eq. (16e), shown in Figure 6. One may observe that when the gravity is absent the vehicle moves with an oscillating velocity between 0 and about 18 m/s, i.e., the mean average is upward of about 9.0 m/s (red line). In contrast, the gravitation causes again an oscillatory vehicle's velocity but now the average value decreases linearly proportionally with the time t (blue line). In the latter case, negative vehicle's instantaneous velocity occurs even for the period $0 < t < 0.9\,\mathrm{s} \cong (\lambda\omega_0 \cos\theta_0 / g)$ (using Eq. (18) and observing Figure 5 and Figure 6) in which the vehicle raises up towards its ultimate height.

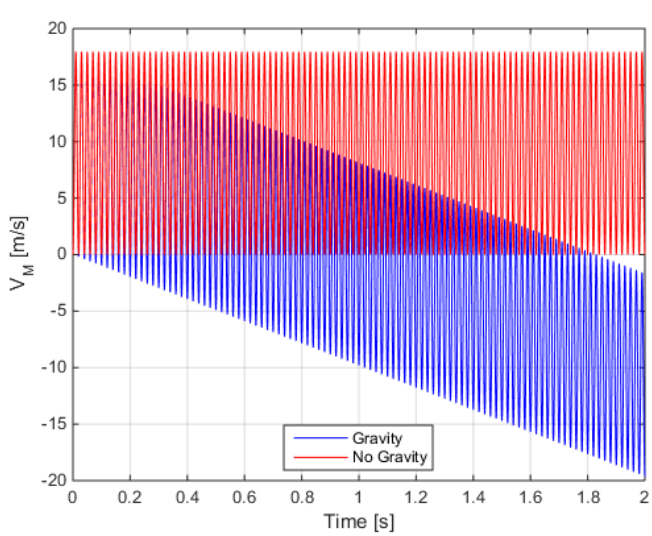

Figure 6. Time history of vehicle's velocity for the first 100 rotations.

Now, we focus on the impulse of the inertial force $F_y = -2m\ddot{y}_{m,rel}$ on both rotating masses in y-direction, without considering the inertial component $-2m\ddot{y}_M$ associated to the accelerated observer (i.e., $y_{m,rel} = r\sin\theta$, instead of the full term $y_m = y_M + r\sin\theta$). In the first quadrant (upper right: $0 \le \theta \le 90^0$) the impulse (i.e., the integral of the force over time) will be:

$$I_1 = \int_0^{T/4} F_y \mathrm{d}t = -2mr \int_0^{T/4} \left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right)\mathrm{d}t \overset{\ddot{\theta}=0}{=} 2mr\omega^2 \int_0^{T/4} (\sin\omega t)\mathrm{d}t \overset{\omega T = 2\pi}{=} \tag{44a}$$

$$-2mr\omega^2 \left[\frac{\cos(\omega t)}{\omega}\right]_0^{T/4} = 2mr\omega$$

Similarly, in the second quadrant (upper left: $90^0 \le \theta \le 180^0$) the impulse will be:

$$I_2 = \int_{T/4}^{T/2} F_y \mathrm{d}t = 2mr\omega^2 \int_{T/4}^{T/2} (\sin\omega t)\mathrm{d}t = -2mr\omega^2 \left[\frac{\cos(\omega t)}{\omega}\right]_{T/4}^{T/2} = 2mr\omega \tag{44b}$$

Similarly, in the third quadrant (lower left: $180^0 \le \theta \le 270^0$) the impulse will be:

$$I_3 = \int_{T/2}^{3T/4} F_y \mathrm{d}t = 2mr\omega^2 \int_{T/2}^{3T/4} (\sin\omega t)\mathrm{d}t = -2mr\omega^2 \left[\frac{\cos(\omega t)}{\omega}\right]_{T/2}^{3T/4} = -2mr\omega \tag{44c}$$

Finally, in the fourth quadrant (lower right: $270^0 \le \theta \le 360^0$) the impulse will be:

$$I_4 = \int_{3T/4}^{T} F_y \mathrm{d}t = 2mr\omega^2 \int_{3T/4}^{T} (\sin\omega t)\mathrm{d}t = -2mr\omega^2 \left[\frac{\cos(\omega t)}{\omega}\right]_{3T/4}^{T} = -2mr\omega \tag{44d}$$

The results are illustrated in Figure 7, where one may observe equal positive impulses in the two top quadrants and negative ones in the two bottom ones. Therefore, the total upward impulse is equal and opposite of the downward one. In other words, the integral $\int_0^t [F_y(t)]\mathrm{d}\tau$ which appears in the right-hand side of Eq. (16f), i.e., the impulse of inertial force, vanishes, and therefore "the net thrust is zero".

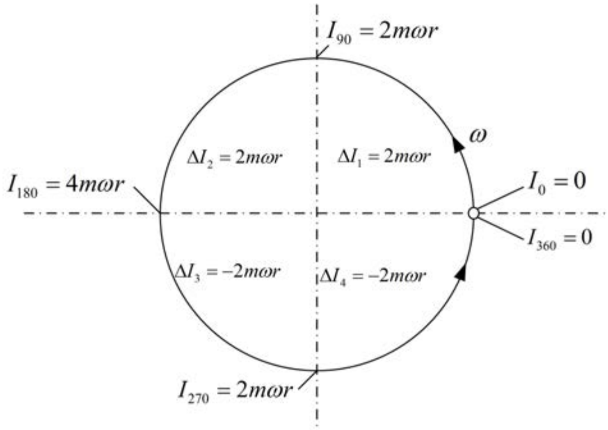

Figure 7. Incremental impulse ΔI of the inertial force in the four quadrants on the vertical plane, and characteristic values I_θ at $\theta = 0, 90, 180, 270, 360$ degrees, induced by the two rotating masses m (constant angular velocity ω).

For any intermediate time instance t, the impulse with reference the initial position ($\theta_0 = 0$ at $t = 0$, and thus $\cos\theta_0 = 1$) is given by:

$$I(t) = \int_0^t 2m\omega^2 r \sin\theta(t)\,\mathrm{d}\tau = \int_0^t 2m\omega^2 r \sin\theta(t)\frac{\mathrm{d}\theta}{\omega} = 2m\omega r\int_0^\theta \sin\theta \mathrm{d}\theta$$
$$= -2m\omega r\left(\cos\theta - \cos\theta_0\right), \tag{45}$$

and its graphical representation is shown by the blue line in Figure 8 (two full rotations). One may observe that -in this case- the impulse is a positive function everywhere except of the initial point of which the conditions are repeated at polar angles $\theta = 0, 2\pi, 4\pi$, etc. In other words, at the end of each rotation, the corresponding incremental impulse of the inertial forces vanishes. Nevertheless, the mean average of the impulse within a period T (rotation of 2π radians)

$$I_{\text{average}} = \frac{\int_0^T I(\tau)\,\mathrm{d}\tau}{T} \underbrace{=}_{\text{Eq.(45)}} \frac{\int_0^T -2m\omega r\left(\cos\theta - \cos\theta_0\right)\mathrm{d}\tau}{T} = \frac{\omega\int_0^{2\pi} -2m\omega r\left(\cos\theta - \cos\theta_0\right)\frac{\mathrm{d}\theta}{\omega}}{(2\pi)} \tag{46}$$
$$\underbrace{=}_{\omega=\text{const.}} 2m\omega r\cos\theta_0$$

is a positive value (see red line) equal to $I_{\text{average}} = 2m\omega r \times 1 \cong 62.8\,\text{N-s}$, which is associated to a linear term which will be discussed below. Therefore, the integral $\int_0^t I(\tau)\mathrm{d}\tau$ is a positive contribution to vehicle's height according to Eq. (17).

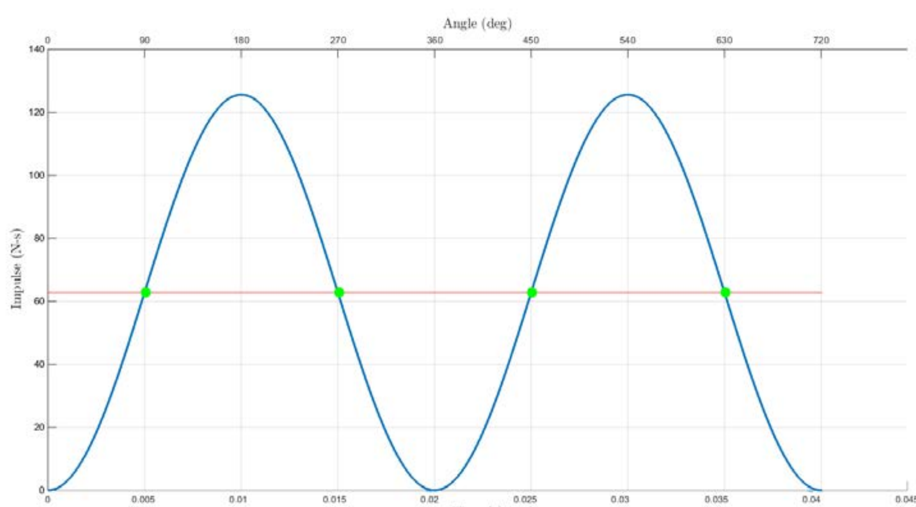

Figure 8. Impulse function of inertial force measured from the initial position ($\theta_0 = 0$ at $t = 0$).

At this point it is instructive to stretch the reader's attention to the fact that the quantity $I(t)=\int_0^t [F_y(t)]\mathrm{d}\tau$ is not that important because it influences only the velocity of the vehicle. From the above discussion it becomes evident that more important is its temporal integral $\int_0^t I(\tau)\mathrm{d}\tau$ which influences vehicle's position, as shown in Eq. (17). At the end, this integral gives the linear term $(2m\omega r\cos\theta_0)t$ plus the small, bounded term $-2mr(\sin\theta-\sin\theta_0)$.

The above discussion was concerned with the case $\theta_0=0$, which is of major importance because is related to the maximum possible linear term $(2m\omega r\cos\theta_0)t$, which leads to the longest travel length (according to Eq. (19)).
For the sake of completeness, let us assume that now $\theta_0=-\pi/2$. The updated results are shown in Figure 9, where one may observe that the average value of the impulse is zero. The latter fact is associated to the incapability of this condition (i.e. $\cos\theta_0=0$) to produce a vertical jump, in accordance with Eq. (17).

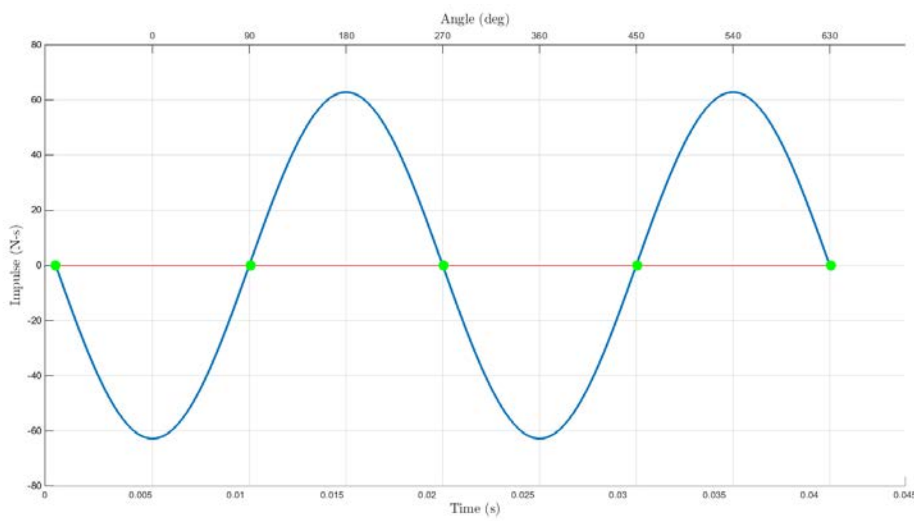

Figure 9. Impulse measured from the initial position ($\theta_0=-90^0$ at $t = 0$).

In both cases ($\theta_0=0$ or $\theta_0=-\pi/2$), the impulse for the first rotation vanishes, and thus

$$\int_0^T I(\tau)\mathrm{d}\tau=0 \qquad \text{or} \qquad \int_0^{T/2} I(\tau)\mathrm{d}\tau+\int_{T/2}^{T} I(\tau)\mathrm{d}\tau=0 \tag{47a}$$

$$\text{whence}\quad \int_0^{T/2} I(\tau)\mathrm{d}\tau=-\int_{T/2}^{T} I(\tau)\mathrm{d}\tau \tag{47b}$$

Therefore, in the case of constant angular velocity the upward impulse is cancelled by the downward one. The open question is whether we can create a non-vanishing impulse per full rotation. Then we must study the consequences.

Exponential angular velocity

Below we study a particular case in which the impulse per revolution is non-zero. We shall merely assume a constant radius r, while the polar angle is controlled to vary exponentially with time

$$\theta(t) = \theta_0 + \frac{e^{at} - 1}{a}\omega_0, \tag{48}$$

whence the angular velocity ω increases as follows:

$$\omega(t) = \dot{\theta}(t) = \omega_0 e^{at} = \omega_0 + a(\theta - \theta_0), \tag{49}$$

where $a = \mathrm{d}\omega/\mathrm{d}\theta = \text{const.}$, while ω_0 and θ_0 are the initial angular velocity and polar angle, respectively. Obviously, in this case the relationship between time and polar angle is

$$t = \frac{1}{a}\ln\left[\frac{a(\theta - \theta_0)}{\omega_0} + 1\right] \tag{50}$$

According to the general formula of Eq. (15f), the impulse of inertial forces is given by:

$$I(t) = -2mr\left(\omega\cos\theta - \omega_0\cos\theta_0\right) \qquad \text{(16g) repeated}$$

Since the angular velocity ω increases with time according to Eq. (49), after a full revolution of 360° (where $\cos\theta = \cos\theta_0$), we have $\omega > \omega_0$, and thus the corresponding impulse of the inertial force will be

$$I(2\pi) = -2mr\left(\omega - \omega_0\right)\cos\theta_0 = -2mra\underbrace{\left(\theta - \theta_0\right)}_{2\pi}\cos\theta_0 = \left(-4mr\pi a\right)\cos\theta_0 \neq 0 \tag{51a}$$

Since according to Eq. (49) $\omega - \omega_0 = +a(\theta - \theta_0)$, at the end of each rotation ($\theta_{end} = 2\pi, 4\pi, \cdots, 2k\pi$) where $\cos\theta_{end} = \cos\theta_0$), the value of the impulse function is given by:

$$I(2k\pi) = \left(-4k\pi ar\right)\cos\theta_0, \quad k = 0, 1, 2, \ldots, \tag{51b}$$

and thus, its algebraic value progressively *decreases*.

Moreover, it is worthy to mention that by virtue of Eq. (16g), the value of the impulse function fulfils the condition:

$$I\left(2k\pi + \frac{\pi}{2}\right) = I\left(2k\pi + \frac{3\pi}{2}\right) = 2mr\omega_0\cos\theta_0, \quad k = 0, 1, 2, \ldots, \tag{52}$$

which means that the change of the impulse between the upper and lower points of the rotating mass is zero, as shown in Figure 10.

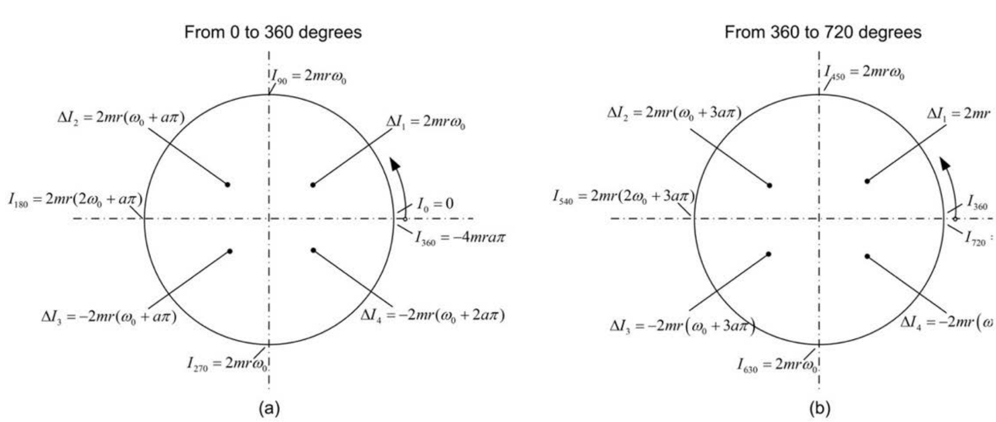

Figure 10. Incremental impulse of total inertial force in the four quadrants on the vertical plane for (a) the first and (b) the second rotation (exponentially increasing angular velocity ω).

Considering initial conditions ($\omega_0 \cong 314.16\,\mathrm{s}^{-1}$, $\theta_0 = 0$) and taking the factor $a = 1$, for the first two rotations the calculated impulse function is shown in Figure 11. Although the angular velocity increases, one may easily find that the average impulse is expressed in terms of the initial value ω_0 by

$$I_{\text{average}}^{\text{exponential}} = \frac{\int_0^T I(t)dt}{T} = 2mr\omega_0 \cos\theta_0 = 2\times1\times0.1\times314.16\times1 = 62.8319\,, \tag{53}$$

In addition, the endpoints of each rotation are shown by green bullets. The impulse function $I(t)$ takes the following negative values: -1.2566 (first rotation), -2.5133 (second rotation). Note that the positive value in Eq. (53) is closely related to the initial velocity of the centre of mass and is the cause of the upward jump. On the other point of view, the nonzero value of the impulse function $I(t)$ at the end of each rotation according to Eq. (51b), shows that the average inertial force, i.e. the integral of the inertial forces over a period is nonzero and this nonzero value progressively increases in absolute value.

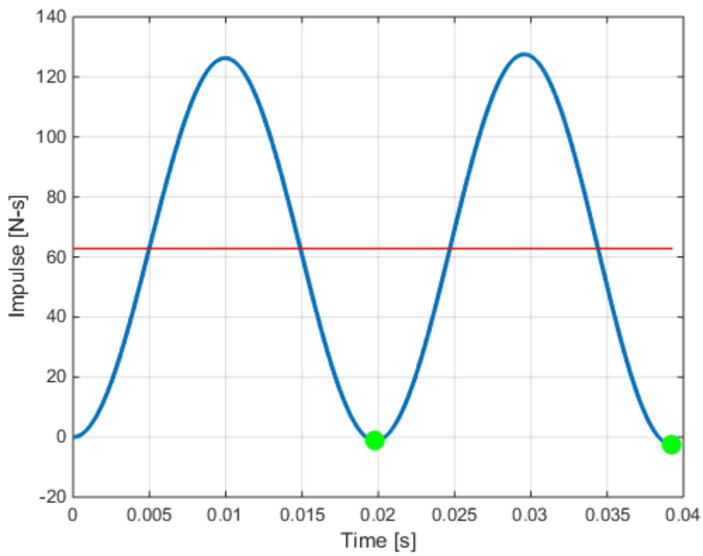

Figure 11. Variation of the impulse function and average value for the first two rotations (exponentially increasing angular velocity ω).

Moreover, to make the situation clear, next we increase the number of rotations to 100. One may observe that the amplitude of the impulse function *I(t)* progressively increases, while at the end of each rotation the impulse function *I(t)* takes the following negative values:
-1.2566 (first rotation),
-2.5133 (second rotation), as earlier mentioned, and also
-3.7699 (third rotation), -5.0265 (end of fourth rotation), until
-125.6637 (end of 100$^{\text{th}}$ rotation), which are marked again by green bullets in Figure12. The average value is again marked by the red line and remains the same even if the function *I(t)* takes negative values as well.

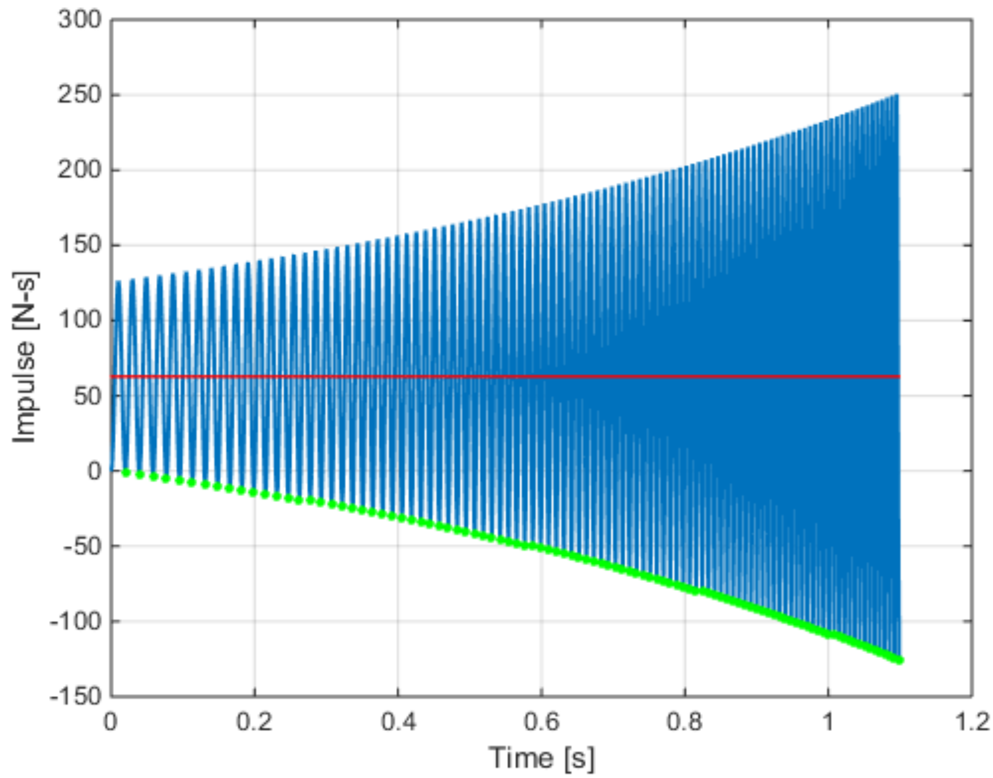

Figure 12. Time history of the impulse function (exponentially increasing angular velocity ω).

Linearly increasing angular velocity

Below we study a second case in which the impulse of inertial forces per revolution is non-zero. Again, we assume a constant radius r, but now the angular velocity ω increases linearly in terms of time as follows:

$$\omega = \omega_0 + at, \tag{54}$$

where a is a constant, and t is the time. If we assume that at the initial time $t = 0$ the rods form the angle θ_0 with the horizontal line, the polar angle $\theta(t)$ will be given by

$$\theta(t) = \frac{1}{2}at^2 + \omega_0 t + \theta_0, \tag{55}$$

and thus, Eq. (55) depicts that the elapsed time t can be expressed in terms of the travelled polar angle θ as the trivial positive solution of the binomial:

$$at^2 + 2\omega_0 t + 2(\theta_0 - \theta) = 0 \tag{56}$$

Substituting Eq. (54) into Eq. (16g), we obtain the impulse function $I(t)$ directly in terms of time t:

$$I(t) \equiv \int_0^t F_{inertial} \mathrm{d}\tau = -2mr \cdot (\omega_0 + at) \cos\left(\frac{1}{2}at^2 + \omega_0 t\right) + \underbrace{\left(2mr\omega_0 \cos\theta_0\right)}_{\text{constant}} \tag{57}$$

The cumulative impulse becomes:

$$\int_0^t I(\tau)\mathrm{d}\tau = \underbrace{-2mr\cdot\sin\left(\frac{1}{2}t(at+2\omega_0)\right)}_{\text{bounded}} + \underbrace{\left(2mr\omega_0\cos\theta_0\right)\cdot t}_{\text{proportional to time}} \tag{58}$$

One may observe that Eq. (58) includes a bounded term of amplitude $2mr$ plus the usual term $\left(2mr\omega_0\cos\theta_0\right)\cdot t$. The latter is proportional to time and constitutes the cause of vehicle's upward jump.

Regarding the average impulse, a difficulty arises about the upper bound of time interval $[0,T]$ over which the integral of the inertial force will be performed to determine the impulse $I=\int_0^T F_{inertial,y}(t)\,dt$ in Eq. (57). Since the only period is geometrically determined by the repeated revolutions, a reasonable choice is to consider a full revolution, from θ_0 τo $\theta=\theta_0+2\pi$ (say for $\theta_0=0$). Therefore, in the first rotation the time T_1 which is required to complete (i.e. $\theta=2\pi$) will be:

$$T_1=\frac{-2\omega_0+\sqrt{4\omega_0^2+8a\theta}}{2a}\approx 0.0199\,\mathrm{s}\,. \tag{59}$$

Next, setting $\theta=4\pi$ in Eq. (59) we determine the sum $(T_1+T_2=0.0396\,\mathrm{s})$, whence the elapsed time which is consumed only for the second rotation becomes $(T_2=0.0197\,\mathrm{s})$, i.e. a little smaller than the value $T_1=0.0199\,\mathrm{s}$ found earlier for the first rotation. This means that the period is not constant.

Moreover, one can easily validate that the impulse function at the end of the first rotations is nonzero, i.e. $I(2\pi)=-0.5972<0$, which means that the time integral of the inertial forces is negative.

Taking the average value in the abovementioned time span T_1, we have:

$$I_{\text{average}}^{\text{linear}}=\frac{\int_0^{T_1} I(t)\mathrm{d}t}{T_1}=2mr\omega_0\cos\theta_0-2mr\cdot\frac{\int_0^{T_1}\sin\left(\frac{1}{2}t(at+2\omega_0)\right)\mathrm{d}t}{T_1}=62.8319+\text{Small term}\,. \tag{60}$$

Unfortunately, the second definite integral in Eq. (60) can be expressed only in terms of Fresnel integrals but it is small compared to the first term of the same equality.

Nevertheless, for specific limits the numerical value of this integral is available. In conclusion, for the first revolution of 360° starting from the horizontal direction, the incremental impulse is given in Figure 13.

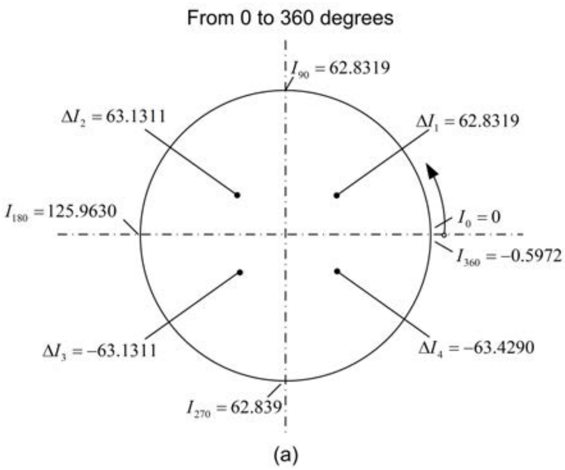

Figure 13. Incremental impulse in the four quadrants on the vertical plane (linearly increasing angular velocity) of two rotating masses, each of magnitude $m = 1$.

One may observe that for every 360° the current impulse is different than zero and its absolute value increases from the first to the second rotation. It is remarkable that the negative value at the end of each rotation is because the bottom values of the inertial force are higher than those of the top values. Obviously, if we change the direction of rotation the signs are reversed.

As also happened in the two previous cases (constant and exponential variations of the angular velocity), the magnitude of the impulse functions is preserved at the upper and lower points, while the impulse increments in the second and third quadrant cancel one another.

Comparison of the three models

The three models (with blue, red and dashed green colour, respectively) are shown in Figure 14 for one hundred rotations and for the same initial angular velocity $\omega_0 = 314.1593\,\mathrm{s}^{-1}$.

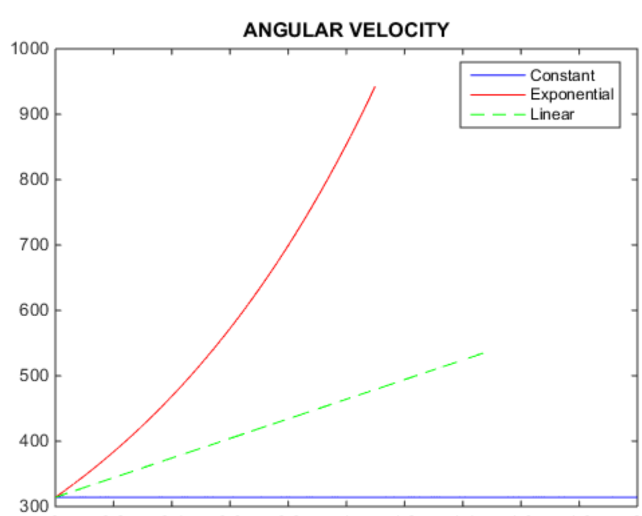

Figure 14. Variation of the angular velocity in the three test cases.

For these three different patterns of angular velocities, Figure 15 shows the corresponding different patterns of vehicle's velocities. Note that in each model, the 100 rotations correspond to different final time span, the largest for the constant and the shortest for the exponential variation of the angular velocity.

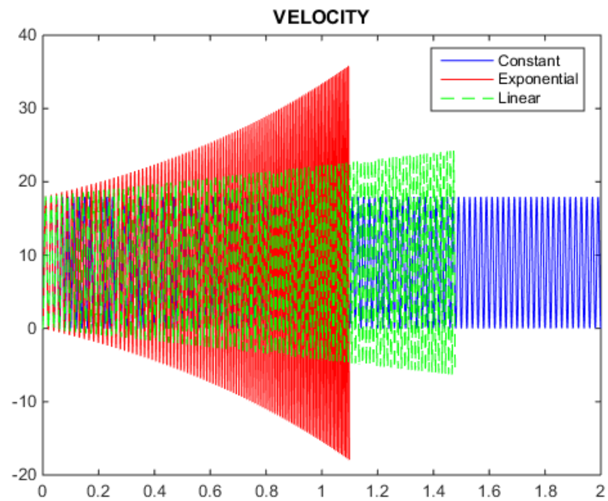

Figure 15. Vehicle's velocity.

The most important finding is probably that, despite the different patterns of vehicle's velocity, no essential visual difference can be observed in Figure 16 regarding the vehicle's lifted height for zero gravity, in which in all three cases the average velocity is 9 m/s. Clearly, when the gravitation is considered, all the three waveforms of the angular velocity $\omega(t)$ are very close to the parabola shown in Figure 5.

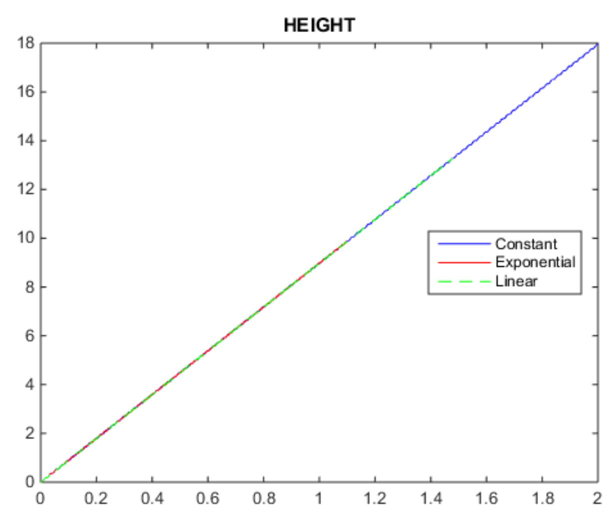

Figure 16. Vehicle's lifted height for the same initial conditions ($\theta_0 = 0, \omega_0 = 314.1593\,\mathrm{s}^{-1}$) and zero gravity.

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Open, closed and quasi-closed systems

While the vehicle sits of the ground surface, the above-described system is *open* since it is based on the surrounding ground surface. Because of this reason, the reaction force F_{reac} (which is part of the total external force $F_{ext} = F_{reac} - (M + 2m)g$) depends on the inertial force. Indeed, according to Eq. (8) we have $F_{reac} = (M + 2m)g - F_{inertial}$. This means that the reaction force (minus the total weights which is also an external force) is continuously dependent on the first temporal derivative of the system's linear momentum.

After the vehicle is detached from the ground, the system continues to be an open system although the reaction force vanishes. This happens because the gravity forces are external and thus their impulse equals to the change of total linear momentum. In this context, as the time passes the velocity of the system during the upward shot decreases until it becomes zero and then the vehicle returns to the ground and remains an open system with a nonzero reaction force. Note that when the (external) gravity could be neglected compared to the inertial force, the system becomes *closed* and thus the vehicle could permanently move upwards.

Contra-rotating masses in the upper half-space

In this section we study another possibility to produce upward impulse of inertial forces, by reducing rotation on only the upper part of a circular path.

Let us consider two contra-rotating masses at the ends of rigid rods of length r, which are moving like car wipers (Figure 17). This means that the masses start from the horizontal position (A_1 and B_2) at a zero velocity and after a rotation of 180 degrees take again the horizontal position (A_2 and B_1) at zero velocity. Obviously, the period $T_p = 2T$ consists of two half-rotations for each mass (see Figure 18). Even though each mass moves only above the center of the associated circle, the time integral of the vertical inertial force component F_y (i.e., the impulse $I_y = \int_0^t F_y(\tau)\,\mathrm{d}\tau = \Delta P$) vanishes in each half period (rotation by 180 degrees), because the change of the linear momentum ΔP (at A_1-A_2 and B_1-B_2) is zero, as a difference of aforementioned zero values at the ends.

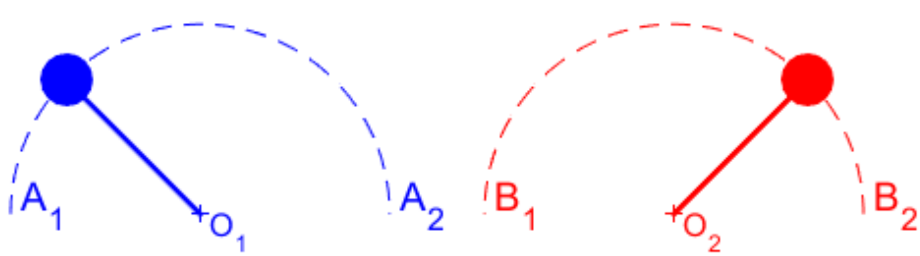

Figure 17. Motion of two contra-rotating masses in the upper plane (from A1-to-A2 and at the same time from B2-to-B1, and *vice versa*).

In addition to the abovementioned physical explanation, here we also provide a mathematical proof for some specific conditions, as simple as possible. Since the two masses are instantaneously still at the extreme positions (A1, B2) and (A2, B1), respectively, each of them will fulfil the following conditions of motion:

$$\text{Points A1 and B2:} \qquad \theta(0)=0, \quad \omega(0)=\dot{\theta}(0)=0 \tag{61}$$

$$\text{Points A2 and B1:} \qquad \theta(T)=0, \quad \omega(T)=\dot{\theta}(T)=0 \tag{62}$$

Without loss of generality, we assume that the polar angle $\theta(t)$ is approximated by a Hermite polynomial, which obviously accurately fulfils the true conditions described by Eq. (61) and Eq. (62), and thus we have:

$$\theta(t)=\left[-2\left(\frac{t}{T}\right)^3+3\left(\frac{t}{T}\right)^2\right]\pi, \quad 0\le t\le T \tag{63}$$

Taking the first derivative of $\theta(t)$ from Eq. (63) with respect to time t, the angular velocity is given by:

$$\omega(t)=\dot{\theta}(t)=6\left[-\left(\frac{t^2}{T^3}\right)+\left(\frac{t}{T^2}\right)\right]\pi, \quad 0\le t\le T\,. \tag{64}$$

Both, the polar angle and the angular velocity, which are given by Eq. (63) and Eq. (64) respectively, are illustrated for an entire period (forward and backward motion) in Figure 18.

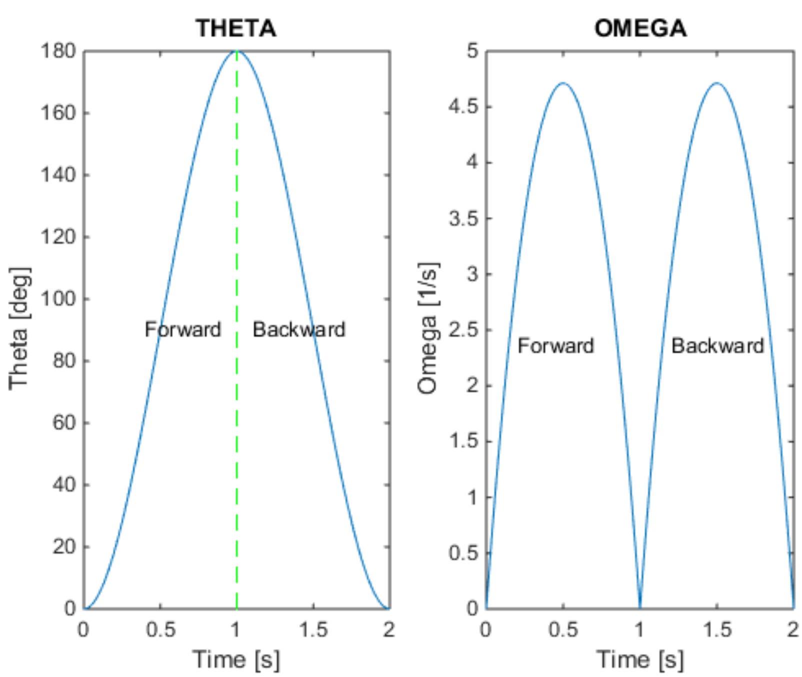

Figure 18. Variation of the polar angle $\theta(t)$ and the angular velocity $\omega(t)$.

Taking the first derivative of $\omega(t)$ from Eq. (64) with respect to time t, we have

$$\dot{\omega}(t)=\ddot{\theta}(t)=6\left[-\left(\frac{2t}{T^3}\right)+\left(\frac{1}{T^2}\right)\right]\pi,\quad 0\le t\le T \tag{65}$$

The total inertial force is given by

$$F_{inertial}=2mr\left(\omega^2\sin\theta-\dot{\omega}\cos\theta\right) \tag{66}$$

Substituting Eq. (63) to Eq. (65) into Eq. (66), we derive a tedious analytical expression for $F_{inertial}$. Moreover, the total impulse of the inertial force for the first half of the entire period (i.e., for $0\le t\le T$) will be given by

$$I_T=\int_0^T F_{inertial}(t)\,\mathrm{d}t=2mr\int_0^T\left(\omega^2\sin\theta-\dot{\omega}\cos\theta\right)\mathrm{d}t \tag{67}$$

Furthermore, if for the sake of simplicity, we consider that $T=1$, Eq. (67) takes the form:

$$I_T=2mr\left(I_1-I_2\right), \tag{68}$$

where, according to MATHEMATICA (www.wolframalpha.com), we have:

$$I_1=\int_0^1\left(\omega^2\sin\theta\right)\mathrm{d}t=\int_0^1 36\pi^2(t-t^2)^2\sin(\pi(3t^2-2t^3))\,dx\approx 8.48067 \tag{69}$$

and

$$I_2=\int_0^1\left(\dot{\omega}\cos\theta\right)\mathrm{d}t=\int_0^1 6\pi(1-2t)\cos(\pi(3t^2-2t^3))\,dx\approx 8.48067 \tag{70}$$

Substituting Eq. (69) and Eq. (70) into Eq. (68), one obtains:

$$I_T=0, \tag{71}$$

Equation (71) denotes that during each half T of the total period $T_p=2T$, the impulse of the total inertial force vanishes.

Discussion

Inertial propulsion has no relationship with perpetual systems. The rotation of the masses may be performed by consuming any kind of energy such as chemical (batteries) or electrical (motor) etc., and thus internal torques and internal shear forces are transmitted to the rigid arms at the joints J_1 and J_2. In this sense, the devotees of the technology cannot be called irrational. Nevertheless, the major mistake of the enthusiasts is that they consider the inertial forces as if these were external ones.

Regarding the famous patent granted by Norman Dean (1959), it is accepted that "Dean drive" consists of two contra-rotating masses of the same constant angular velocity ω which are attached to a vehicle. In the present paper, it was clearly shown that, under the condition of constant angular velocity, this drive creates an upward impulse of inertial forces in the first half rotation ($0\le\theta<\pi$) which is fully cancelled by the downward one in the second half ($\pi\le\theta<2\pi$). Nevertheless, despite this null impulse per rotation, it was shown that it is possible that the Dean drive can move the vehicle on which it is attached, provided a nonzero cosine of the initial arms' polar angle ($\cos\theta_0\ne 0$). This is strongly related to a nonzero initial velocity of the center of mass.

Due to the abovementioned obvious cancellation of the impulse of inertial forces within any full rotation, considering the inertial (d'Alembert) fictitious force as an external one, many inventors have tried to modify Dean's concept in many ways with the utmost desire to strengthen the upward part and lower the downward one, thus allowing a supposedly permanent lift of the vehicle from the ground surface. As they have claimed, one way is to modify the shape of the lower half of the circle on which each mass rotates (see, for example, Figure 19, from Robertson, 2001). The main idea is that by shortening the lower part, the upward impulse will dominate, and thus the vehicle will jump upwards. This issue was discussed in section “Non-circular path”, when was shown that the variation of the radius does not offer better conditions for inertial propulsion.

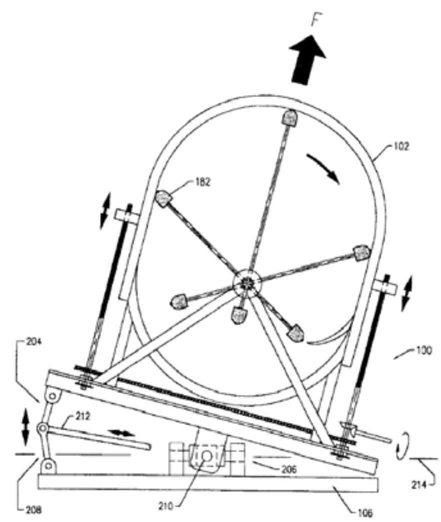

Figure 19. One way to modify the lower radius in the path of the eccentric masses (from Robertson, 2001).

Interestingly, there is a simple mechanical analogue of Dean drive which may be easily understood by everyone. It was shown that Dean drive is very similar to a spring-mass system. Let us consider a vertical linear spring AB, which is supported on the ground surface at the lower endpoint A, while a concentrated mass m_s is attached to the upper endpoint B. The spring is compressed at point B by length Δy and thus a compressive force of magnitude $F_{\text{support}} = k\Delta y$ appears throughout the spring. If the compressive force is much larger than the weight of the mass, i.e. $k\Delta y > m_s g$, and then the system is left free to move (i.e. the compressive force at B is suddenly released), one can easily understand that the spring will be progressively released and when after a very short time will obtain its initial undeformed length will push the ground with no instantaneous reaction force, with the mass at B having its maximum velocity v_0. Then, it is obvious that the system will jump upwards performing a vertical shot of travel $h = v_0^2/(2g)$, where g is gravitational acceleration. In other words, we could say that a ‘Dean drive’ simply replaces and is equivalent to the concept of the catapult.

Another way to supposedly bypass the zero impulse is to introduce unidirectional degrees of freedom at the centers of the circular paths on which the two mass rotate (i.e., by manufacturing flexible joints). In this way it was hoped that an artificial push will be possible when the impulse is directed upwards (Hampton 2022). Other inventors apply the principles of Control Theory to achieve net thrust, as they claim (Gutsche 2018a,2018b). This paper has shown that shifted joints and other kinds of periodic motion do not contribute (see section “Shifted joints”).

Although many of the above aspects have been already studied during the past 60 years (for a review see: Provatidis, 2024), there were still some gray zones which were resolved in the context of the present paper.

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It was clearly shown that it is possible the impulse of the inertial forces per rotation be different than zero. This factoid appeared in two cases that the angular velocity is not periodic but increased exponentially or linearly with respect to the passed time. Nevertheless, the change of the impulse of the inertial forces between the lowest ($\theta = -90^0$) and the highest point ($\theta = +90^0$) of the moving masses always vanishes because at both these points the normal projection of the velocities in the vertical direction vanish. This obvious observation is also reflected to the term $\omega r \cos\theta$ which appears in Eq. (16g), because at lowest ($\theta = -90^0$) and the highest point ($\theta = +90^0$) we have $\cos\theta = 0$.

The case of periodic rotation of the masses (e.g., piston-crank mechanism, three-bar mechanism, etc.) is much simpler because at the end of each period the same kinematic conditions hold (same ω and same θ), and thus the change of linear momentum vanishes. By this observation we also cover the case of the previously mentioned flexible joints.

A monotonically increasing angular velocity ($\Delta\omega > 0$) causes a negative impulse, while a decreasing one ($\Delta\omega < 0$) causes a positive impulse per rotation. The results of this paper showed that the increasing angular velocity ($\Delta\omega > 0$) did not make the vehicle's motion worse than the constant one, and the same is anticipated for the decreasing one (the case $\Delta\omega < 0$ is left to the interested reader as an exercise).

Moreover, in the present paper it was shown that not even the motion of the rotating masses restricted *in the upper part* of a circle (like contra-rotating wipers) can lead to net upward thrust and impulse. This happens just because the same kinematic conditions appear at the extreme points of the repeated track. In the relevant section "Contra-rotating masses in the upper half-space" of this paper, for the sake of simplicity we considered a simple case of Hermite like variation of the polar angle along the entire upper part. Of course, in real life there will possibly be a flat curve of almost constant angular velocity which will tend to zero near the extreme points of the tracking curve. Nevertheless, such a more realistic approximation will not add something new in the conclusions of the above-mentioned section.

Overall, the findings of this paper suggest that:

- In the absence of external forces from the surrounding environment, the only case in which a permanent overshoot of a vehicle can occur is that of continuous mass ejection.
- When the angular velocity $\omega(t)$ increases or decreases with time, the impulse per rotation becomes negative or positive, respectively [see, Eq. (16k): $I(T) \equiv I(2\pi) = -2mr\Delta\omega\cos\theta_0$].
- When the floor is pulled, the three cases tested, with the same initial velocity, give the same lift of the center of mass (C.M.) of the system.
- The waveform of the function $\omega(t)$ simply changes the oscillating velocity of the vehicle, but not the average lift. It just leads to a minor effect around the C.M. [see: the term $\lambda\left(\sin\theta - \sin\theta_0\right)$ in Eq. (17)].
- The difference between $I(T) = 0$ and $I(T) \neq 0$ influences vehicle's motion through its velocity $\dot{y}_M$ [see Eq. (16l)].
- The inertial force $F_{inertial}$ controls the difference of accelerations between the C.M. and the vehicle. But because C.M. moves in a fixed trajectory and has a standard height and speed versus time, the change of $F_{inertial}$ reflects the change in the speed of the vehicle, but not of the C.M. (see Eq. (30).
- The impulse of inertial forces is given by the formula $I = \int_0^t F_{inertial} dt = (M+2m)\int_0^t \ddot{y}_M dt + (M+2m)gt$ [combination of Eq. (28) and (30)]. Since the term $(M + 2m)gt$ is invariable, every change in the impulse $I = \int_0^t F_{inertial} dt$ leads to a change of $\int_0^t \ddot{y}_M dt$, that is of vehicle's velocity.

- Zero impulse per rotation, $I(T)=0$ with $\omega=\omega_0=\text{const.}$ ($\Delta\omega=0$), means that $(M+2m)[\dot{y}_M]_0^T+(M+2m)gT=0$ (see Eq. (16e)), which implies that $\boxed{\dot{y}_M(T)-\dot{y}_M(0)=-gT}$, i.e., the change of vehicle's velocity per rotation (of period *T*) equals the constant quantity –*gT*.
- In contrast, impulse $I(T)>0$ means that $\boxed{\dot{y}_M(T)-\dot{y}_M(0)>-gT}$, while impulse $I(T)<0$ means that $\boxed{\dot{y}_M(T)-\dot{y}_M(0)<-gT}$.
- In any system it is the total external force which accelerates the C.M.
- To the question "how is it possible to propel the vehicle upwards while the inertial force is not an external force", we answer as follows. When the vehicle is supported on the ground the system is *open*. The C.M. rotates on a fixed orbit but changes speed with the help of an internal torque derived from energy consumption. This fact is somehow like the movement of the human arms resting on the ground, which is possible because of the *support* implied by the existence of an open system (i.e., a part of the human body is fixed to the ground). Therefore, as $\omega(t)$ increases, the maximum upward inertial force also increases, with the result that the support reaction decreases ($F_{reac}=(M+2m)g-F_{inertial}$, with $F_{inertial}=2m\omega^2 r\sin\theta$). Thus, when the upward inertial force $F_{inertial}$ overcomes the total weight of the system, then the support reaction becomes zero and the vehicle detaches from the ground surface.
- Since the above use of the concept of inertial force overcoming weight may raise doubts, we can resort to an even lower level of argumentation. As the two masses rotate, each mass receives a force T from the respective connecting arm in the direction of the center of rotation (J_1 or J_2), which is equal to the corresponding centripetal force ($T=m\omega^2 r$ according to Newton's 2nd law), and then its reaction ($-T$) is transferred through the arm to the vehicle in the radial direction from the center of rotation (J_1 or J_2) to the rotating masses. Thus, overall, the vehicle receives the component of the two forces ($-T_1,-T_2$), which is directed towards the y-axis and is given by the relationship $T_{tot,y}=2m\omega^2 r\sin\theta$. In other words, beyond all doubt, *during the time that the vehicle is attached to the ground*, the vehicle undertakes the inertial force $F_{inertial}=2m\omega^2 r\sin\theta$ as it is.
- During the potential increase in angular velocity $\omega(t)$, apparently the linear momentum of the system is *not* conserved, until the moment the vehicle leaves the ground. But once the vehicle leaves the ground, the system changes state and becomes *quasi-closed*, in the sense that the development of an external reaction force is no longer possible (at least until the vehicle returns to its original level). The word "quasi-closed" refers to the existence of the gravity field which is an external effect and therefore does not actually allow the consideration of a completely closed system; however (as we saw in Figure 4) in a couple of rotations the effect of the gravitation is relatively small.
- The above consideration of the quasi-closed system allows us to understand even better the mechanism of lifting the vehicle. In principle, the maximum lift length occurs when the initial velocity of the center of gravity is maximum. This happens when the two connecting arms are horizontal ($\theta_0=0,\cos\theta_0=1$) so the two point velocities are directed exactly towards the y-axis (momentum $P=2m\omega r$), while in any later time we would get (momentum $P=2m\omega r\cos\theta_0$). Then, after a 90-degree rotation, the arms become vertical and thus the resultant velocity on the y-axis becomes zero, so due to the quasi-closed system the change in linear momentum is taken over by the vehicle.
- In other words, the main reason for the potential lift of the vehicle lies in the sufficiently large initial velocity of the C.M. in the y-direction due to the circumferential velocity of the rotating masses. This speed completely defines the maximum height, as if it were a vertical shot of a material point mass of magnitude $m_{tot}=M+2m$.

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- While the inertial force is completely transferred to the vehicle when it is *on* the surface of the ground (open system), the same cannot happen in the quasi-closed system that results after the vehicle is lifted. In particular, the zero support-reaction dictates that according to Newton's 2nd law (therefore according to the principle of conservation of momentum) the relationship $\ddot{y}_M = F_{inertial}/(M+2m) - g$ applies (see Eq. (16c)), which concerns the acceleration of the vehicle and has been commented on above.
- If after reaching the maximum height ($y_{M,\max} = V_{CM}^2/(2g)$) we wish to push the vehicle further upwards, we would have to 'support' the vehicle on an artificial ground until the arms take (at optimum) their horizontal position and then withdraw this support base. Then we would have a repetition of the same procedure, that is, the vehicle would perform an additional lift again of magnitude $y_{M,\max} = V_{CM}^2/(2g)$, and so on.

The motivation of the present paper was to answer to the question whether inertial propulsion in vacuum is possible or not. Therefore, not much was written about the practical usefulness of the inertial propulsion, a topic that has been discussed in a recent review paper (Provatidis, 2024). For example, the proper utilization of rotating masses may replace the use of a mechanical catapult for vehicle launch, since the required elastic strain energy may be directly substituted by the initial velocity at the center of mass of the inertial drive. Regarding other terrestrial applications, it can be used for control applications such as micro-positioning as well as for space exploration in the sense of a Mars rover.

A weakness of the present paper is that it did not study gyroscopic systems. One useful application based on the conservation of the angular momentum, is the attitude control, in which the orientation of satellites (attitude control) may change simply by changing the shape of rotating gyroscopes, thus avoiding the use of small-scale liquid rockets. Furthermore, regarding the desired net thrust, several patents have claimed to realize it using gyroscopic inertial propulsion (see, e.g., Fiala 2007, 2011, among others). Nevertheless, again the motion of the center of mass depends on the initial conditions, and particularly on the initial velocity. Therefore, the major core of this paper regarding the motion of a vehicle equipped with a gyroscopic drive is still applicable.

Conclusions

In principle, the inertial forces which are induced in mechanical systems by rotation may be capable of producing motion of an object, when out-of-balance eccentric masses exist therein. This may happen because the eccentricities are related to velocities which also influence the initial velocity at the center of mass, which in turn may perform a vertical upward shot. Unfortunately, when the upper point of object's height is reached and the object operates in vacuum without any mechanical support, there is not an obvious technical manner (without expelling material) to maintain the vehicle immobilized (supported) until the eccentric masses are brought at their initial position, and then perform a second upward shot. Mechanical systems which can produce non-vanishing impulse are possible, but the passage of the rotating masses from the lowest and upper positions ($\theta = \pm 90^0$) reduce it at a small amount developed within say 90 degrees, which is bounded and thus in no case is capable of cancelling the continuously increasing quantity –*gt*. Periodic inertial systems induce the same kinematical conditions per revolution, which means vanishing impulse of inertial forces per rotation. While inertial propulsion is of very restricted use in vacuum, it is applicable and may be useful in frictional environments, such as on the ground (based on the stick-slip phenomenon) or in the water.

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