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An epidemical model with nonlocal spatial infections

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Appendix A

Evolution dynamics of the spatial SIR models

A.1 *A 1d example with co-localized initial densities*

In this section, we show more of the numerical simulations of the spatial SIR models for both 1d and 2d cases. We examine the evolution dynamics of a 1d example with the co-localized initial densities:

$$I_0(x) = S_0(x) \propto \exp[-(x-2.5)^2]. \tag{A.1}$$

The initial densities are different from the previous case in Figure 3, where the centers of the two initial densities are significantly separated from each other.

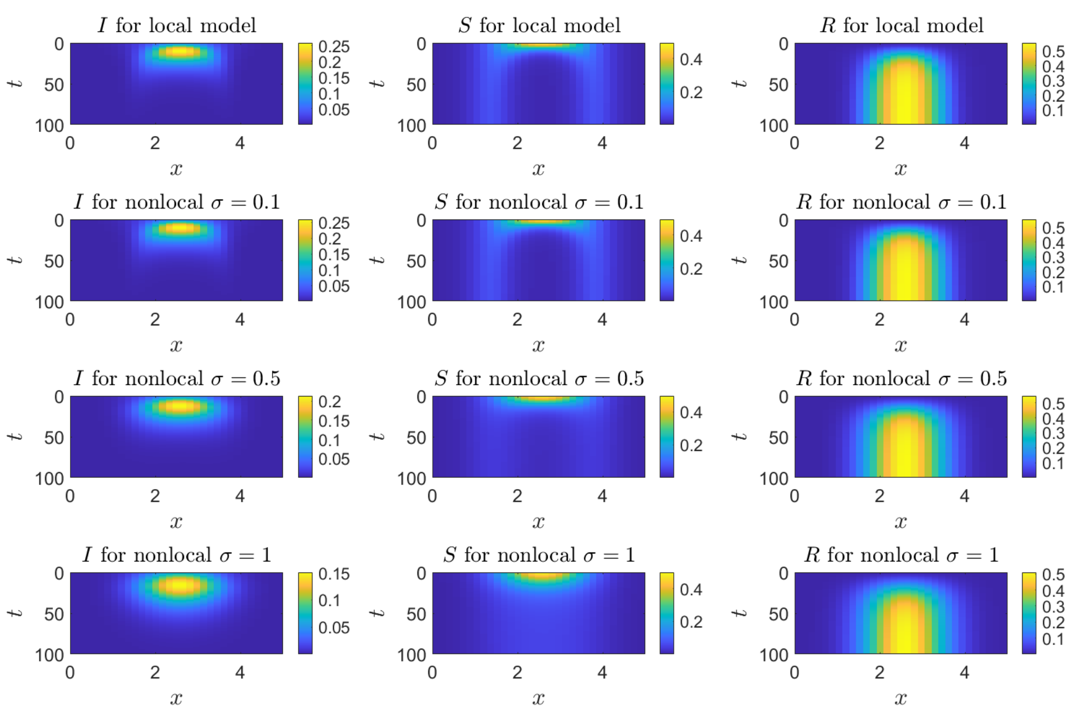

Figure 8. Evolution dynamics of the 1d SIR models with co-localized Gaussian initial conditions (A.1). Across all simulations, we fix $\beta = 0.8$, $\gamma = 0.1$, and $\mu = \frac{1}{2}\beta\sigma^2$. We use $\sigma = 0.1$ for the local model (top panel) and use $\sigma = 0.1$, 0.5, and 1 for the nonlocal models (bottom three panels).

We show the evolution dynamics of the spatial local and nonlocal SIR models in Figure 8. For the local model and the nonlocal model with $\sigma = 0.1$, we notice waves of infection moving away from the mass center at $x = 2.5$. However, for the nonlocal models with $\sigma = 0.5$ and $\sigma = 1$, we observe a diffusion of infection. Additionally, in the local model, the susceptible population shows a low-density region near $x = 2.5$ for $t > 20$. This occurs because, in the local model, infected individuals are more likely to infect their nearby individuals, leading to significant infections and a reduction in population near $x = 2.5$. On the other hand, the nonlocal model with $\sigma = 1$ exhibits a more homogeneous population distribution after $t > 20$, meaning that some susceptible individuals around $x = 2.5$ remain uninfected until the pandemic ends. Infected individuals are less infectious to their neighbors compared to the local models, so some susceptible individuals never get infected before the nearby infected individuals near $x = 2.5$ recover.

A.2 *A 2d example with Gaussian initial densities*

We investigate a 2d local SIR model and plot the iso-surfaces of three densities in Figure 9. The initial densities for the susceptible and infected populations are:

$$
\begin{aligned}
&S_0(x_1, x_2) \propto \exp[-(x_1 - 2.5)^2 - (x_2 - 2.5)^2], \\
&I_0(x_1, x_2) \propto \exp[-(x_1 - 1)^2 - (x_2 - 1)^2].
\end{aligned} \tag{A.2}
$$

The susceptible population has a Gaussian initial density. As shown in Figure 9a, the susceptible population decreases over time, with the center decreasing faster than the perimeters. In the local model, infected individuals are more infectious to their neighbors compared to the nonlocal models. This leads to a rapid spread of infection in high-density regions, causing a faster decay in the center population.

Additionally, since infections are initially located near $(1, 1)$, the susceptible population near $(1, 1)$ gets infected sooner compared to regions farther away, forming an asymmetric pattern in the susceptible iso-surface. The infected population initially increases, but around $t = 1$, the recovery process begins to dominate, leading to a decrease in the infected population.

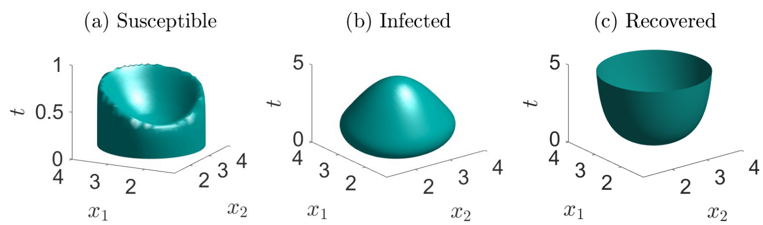

Figure 9. The iso-surface representation of the evolution dynamics of the 2d local SIR model (2.6). The infection rate $\beta = 100$, the recovery rate $\gamma = 0.5$, $\mu = \beta\sigma^2/2$, where the width parameter $\sigma = 0.1$, and the initial infection ratio $\eta = 0.01$. The three population densities ($S(x, t)$, $I(x, t)$, and $R(x, t)$) have constant function values $0.054, 0.038$, and 0.045 on their respective iso-surfaces.